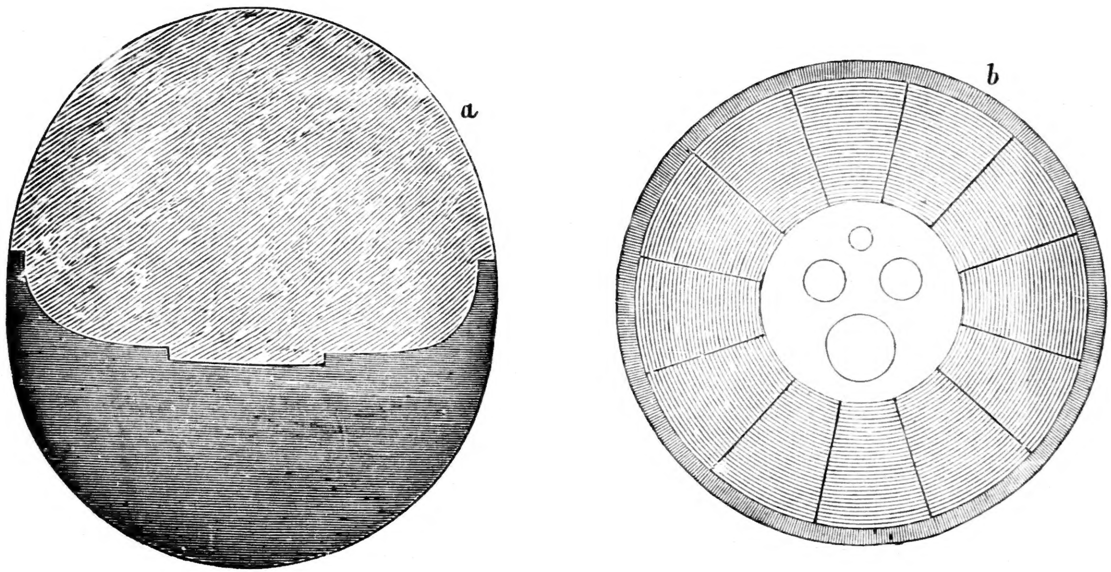
CENTRIFUGAL FORCE AND GRAVITATION

BY

(KUKLOS)

JOHN HARRIS.



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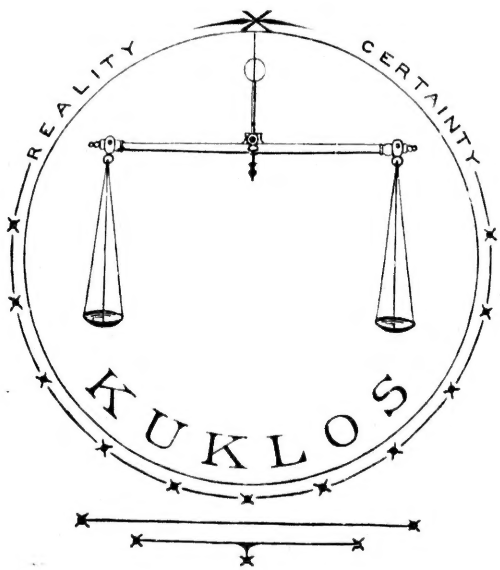

CENTRIFUGAL FORCE

AND

GRAVITATION.

A LECTURE.

Vol 1

BY

JOHN HARRIS.

MONTREAL:
JOHN LOVELL, ST. NICHOLAS STREET.

1873.

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PREFACE.

The 'inductive system' as taught by Francis Bacon bases science upon facts. The necessity of correctly observing and of verifying with the most scrupulous care those fundamental facts upon which the various divisions of natural science are (have to be) constructed, is therein inculcated and insisted upon; the slow and uncertain progress of science when 'undemonstrated theories' and 'suppositious facts' are substituted for 'truths and realities' is pointed out and dwelt upon; the pernicious effect of accepting conclusions resulting from reasoning not *based on truth or certainty* as 'scientific' is plainly shown; and the dangers (and the possibly disastrous consequences) of systemising unsound knowledge and arraying falsehood in the garb of truth are clearly indicated.

The primary purpose of this book is to ascertain whether the rules of the inductive system as laid down and taught by Francis Bacon are still recognized; that is to say, whether the number of those who do recognise those rules is sufficient to compel their public and general recognition as constituting, collectively, that *law of science* which may not be set aside by any human author-

ity,—that law to which each individual, if educated and qualified, has the right (the most important of all rights which freedom can give) to appeal in his own interest and that of others; and, that law according to which his appeal must be heard, and by which the soundness or unsoundness of his claims, must be decided and determined.

With this object certain important doctrines now taught as belonging to science are herein contested on the general ground that the specified conclusions so taught as facts are false or unsound; the precise nature of the error being pointed out in each case, and the correct or sound explanation set forth. The subjects relate more particularly to what may be termed the Physics of Astronomy. Of these may be instanced:

The Theory known as the	'Newtonian law of Gravitation.'
" " " "	'Kepler's third law.'
" " " "	'The Law of Equable Areas.'

The Teaching on the subject of the tides.

The present teaching on these subjects is challenged, according to the rules of the inductive system, *on fact and for cause shown.* For example—The distance of the sun (i. e. the approximate distance) from the earth, as ascertained by parallax, is an astronomical fact; also the moon's distance from the earth, and the periodic times of the moon's orbital revolution round the earth, and of the earth's revolution round the sun, are astronomical facts. The attractive force of gravitation at the surface of the earth as measured by the space fallen through in a de-

finite time by an attracted body, is a fact belonging to physical science. It is asserted for reasons particularly set forth that the theory known as the Newtonian law of gravitation is not based upon and is irreconcileable with these facts; and the theory (alleged to be) correctly based upon these facts and demonstrated by them to be sound, is stated and explained.

Our statements and arguments are put forward in such a form that if erroneous or inconclusive they may readily be shown to be so. If a deficiency as to a proper knowledge of the subject (or subjects) is apparent, that may be easily pointed out. If it is said that we have not in this investigation employed the analytical methods of what are called the higher branches of mathematics, we acknowledge that we have not done so; but we are not bound or called upon by the rules of the inductive system to treat the subject in such a manner, and we are decidedly of opinion that these subjects being of a fundamental and primary character, such a method of treatment would be improper and unsafe.

MONTREAL, 28*th June*, 1873.

LECTURE.

CENTRIFUGAL FORCE AND GRAVITATION.

INTRODUCTORY OBSERVATIONS.

BEFORE commencing my formal lecture it will be proper to give a brief explanation of the particular purpose and of the reasons which I trust will be held to justify me in coming forward as a public teacher or lecturer on a scientific subject. It is my intention to make some general observations either at the close of this lecture or soon afterwards, on obstacles to the progress of science, having particular reference to certain scientific difficulties and questions in controversy on scientific subjects. I should have preferred to defer all explanation until that time, and to commence at once upon the particular subject of the lecture, but so doing would certainly expose me to the risk of a prejudice being formed in your minds, antagonistic (and possibly very strongly antagonistic) to my purpose as an instructor or lecturer. Under the circumstances this could be only a temporary obstacle, but an unfavorable prejudice is in itself an obstacle of a kind which I am aware that a teacher should by no means despise or underrate; and, as the subject has already sufficient difficulties of its own, it is requisite for me to guard, as carefully as I can, from unnecessarily increasing them. The subject of my lecture (Centrifugal Force and Gravitation) may be considered to belong more directly and

particularly to that division of natural science called mechanics, or mechanical philosophy; but also, since it embraces those laws by which the motions and relative positions of the planetary bodies are determined and regulated, it enters largely into that division termed astronomical science. The express purpose of my lecture is to call your particular attention to and clearly explain, so far as may be necessary, the teaching on this subject at present considered scientific (i. e. scientifically orthodox), in order to object, on scientific grounds and for reasons which will be particularly stated, to certain parts of that teaching. The form of the argument will be to give first that which I assert to be the correct and sound teaching on each of those parts of the subject, and then to compare these with the present authorised explanations (i.e. the teaching now recognised as scientific) in order to show and bring distinctly under your consideration the particulars against which my arguments will be directed, as being erroneous and as belonging therefore to unsound science. In making the few preliminary remarks which seem to be necessary I am anxious to avoid making any statement or assuming anything which may even appear to be objectionable or open to dispute. For the moment, it may seem that I am at least risking something in such a sense by proposing to assume that there is in the minds of a great many educated people a conventional, loose, unjustifiable and incorrect meaning attached to the word 'Science,' and therefore also to such compound terms as 'scientific teaching' and 'scientific authority.' I feel sure that I may safely go further than this, and still for the moment only risk dissent, in assuming that this conventional and incorrect meaning is not confined to the minds of persons of good general education, but also finds a place in very many minds which have had the advantage of what is termed a special scientific training—in the minds of men who it may be are quite qualified (possibly much better qualified than myself) to give a correct definition of, and

therefore to attach a correct meaning to the expression 'Science,' but who in fact go on retaining and subjecting themselves to the influence of a more or less indefinite and incorrect meaning attached to that expression. 'Science' is not a new word—it has been in use for at least several centuries, and has been used all that time to convey the same general idea or meaning. The way in which the term has been and still is used, may be described as a definite word used indefinitely; or, to amplify this a little, a word or expression which is essentially definite and discriminating used indefinitely and for the most part so loosely as to be allowed to include things not only dissimilar but even such as are opposed to each other, in the sense that *truth* is opposed to *untruth*, or as right is opposed to wrong. Let us go back only one century: and, with our present advantages and better means of forming a correct judgment, reflect whether all the knowledge which was at that time considered or classed as scientific knowledge was correctly entitled to be so classed or considered. It is beyond question and dispute that opinions, judgments and conclusions were held by and formed a part of the so-called scientific knowledge of the men of science of that day which at the present time would be unhesitatingly condemned by any educated person as being certainly erroneous; and this misapplication of the word was not peculiar to that or to any other one period of the world's past history; nor has it ceased to be still used in essentially the same way, that is misapplied; a source of mischief and confusion resulting from a tacit agreement to deceive ourselves and to call a thing *that* which every one qualified to judge knows that *in fact* it is not; namely, to call a collection of knowledge, some of which is undoubtedly true, some of which is almost certainly or very probably true, some of which is doubtful (i.e. possibly true and possibly untrue), and some of which is certainly untrue, (because no sane scientifically educated man will assert that all which we

now call science contains no errors, no doubtful theories, or merely plausible assumptions held as possible truths until more certain knowledge can be obtained)—to call such a collection collectively by the name science, a word expressly meaning true knowledge as distinguished, not only from unsound knowledge, but also from knowledge not known to be certainly true. The mistake may be partially rectified by adopting some such expression as 'authorised science' or 'classified science,' and defining this to include, together with the scientific knowledge, a certain indefinite amount not certainly known to be sound; but to apply even such a modified expression correctly, a careful re-classification and separation would be necessary; because the present collection, known as science, includes also the great antagonist of science—that evil and false knowledge which, assuming a systematic and apparently scientific form, and entering in unnoticed, mingles with and contaminates knowledge otherwise wholesome and true; and which having insidiously and firmly established itself in the educational stronghold of civilization displays that hatred of definite and true knowledge—that organized opposition to real progress, and that skilful and unceasing endeavor to darken, confound and destroy the human intellect which has ever been the characteristic of 'unsound science':

It may be remarked: well,—as to all this,—there may be something in what you've been saying in a philosophic sense, a sort of abstract truth perhaps; but it can't be practically a matter of any particular consequence, and can scarcely be considered as more than a sort of quibble about the meaning of a word.

Feeling sure that an inevitable controversy is impending upon this particular question—a controversy that will not be confined to a few disputants—not to any one particular locality, nor even to any one nationality, but—a controversy which will become wide-spread, into which all those belonging to the educated world will be drawn, and

in which each one will have to choose his side and take a part—a controversy which will become a conflict of the most uncompromising character, a verbal battle as to which even those who most love peace and detest discord, will agree with those to whom dispute and strife are less unwelcome, that it must be fought, and must be fought *out*, until the one party or the other is completely vanquished and subdued;—with the conviction that such a controversy is at hand, if indeed it may not be said to have already commenced, it would be unadvisable to enter now into an argument the merits of which could not be fairly stated without occupying a good deal of time. Intending, however, as already stated, to make some remarks 'upon obstacles to the progress of science' at the close of this lecture I may then perhaps have something further to say upon this particular subject. Meanwhile I will suppose the foregoing statement or some other equivalent statement to have been made; namely, to the effect—that it is not of practical importance to discriminate, and to define the difference between 'science' (*i.e.* sound science) and 'unsound science'; and I will for the present content myself with asserting the directly contrary, namely—that it is of practical importance so to discriminate—that it is, in an educational sense, of an importance which can be neither overstated nor overrated, to define the difference and to distinguish between 'science' and 'unsound science.'

Before commencing my formal lecture, I beg to state for the benefit of those who take an interest in such subjects, that it will be continued to-morrow evening, as there will not remain much more than sufficient time this evening to make a commencement and to indicate the particular form of the argument. Although I may venture to say that the statements and explanations of my lecture will be conveyed in the most simple and intelligible form that scientific treatment of the subject will justify, it cannot be expected that those who have

not some previous knowledge of the subject will be able to follow with much interest the reasoning and conclusions in an argument which will mainly consist in contrasting and distinguishing between cases essentially dissimilar, but which to the scientifically uninstructed must appear very much alike. I will therefore now take the opportunity to express my personal thanks to those who may this evening have attended in consequence of particular invitation; the lecture itself having commenced those who remain or return will then do so knowing what they have to expect. Of the remarks with which it is intended to conclude the lecture, and which may perhaps be sufficient for an evening to themselves, due notice will be given.

CENTRIFUGAL FORCE AND GRAVITATION.

We will first take three cases for consideration in which a body revolving round a centre (a central body or central point) is subjected to conditions differing as stated.

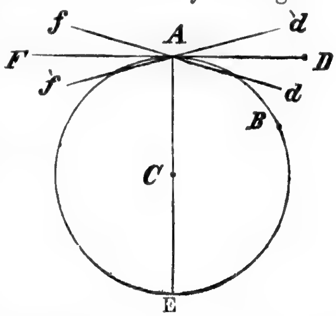
Case 1.—A body attached to a central fixed point revolving in a circle.

Case 2.—A body revolving in a circle with a definite uniform velocity around a central body, the two bodies being subject to the influence of gravitation.

Case 3.—A body retained by and subject to the influence of gravitation, revolving around a central body, the distance of the moving from the central body, being determined by the relative proportions of the velocity and the amount of gravitating force.

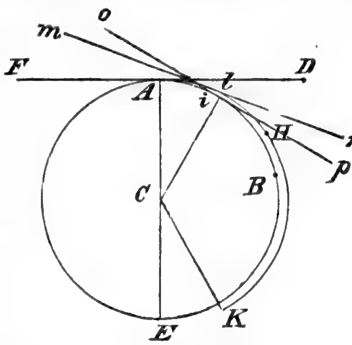
Case (1) may be conveniently subdivided into the case (a) where the revolving body is attached to the fixed central point by a non-elastic line or link; and the case (b) when attached by an elastic line or by a line connected with a spring such as to allow of limited extension when acted upon by the centrifugal force of the revolving body.

(a). Fig. 1. Let a weight A be attached by a string to a central point C, around which it revolves, the motion of A, from the point A, in the direction AD, would carry it to the point D, in a definite time (t). But the motion of A being restrained by the attachment to the central point, is compelled to take place through an arc of the circle AE. To what distance from the point A on the circle will the weight A move in the definite time (t) in which it would, if unrestrained, have moved to the point D, on the straight line AD? Let a point B on the circle AE be the point at which A will arrive in the time (t). Is AB greater or less than, or equal to AD? If greater, then is the motion accelerated; if less, it is retarded; and if equal to AD, then is the motion neither accelerated nor retarded by the action of the string. As there is no ground whatever for assuming that any motive or active force can develope itself from the central point or out of the string, there is no accelerating cause; no extraneous or additional force from which additional motive power or velocity can be derived, therefore AB cannot be greater than AD. Is the motion of A, (leaving out of consideration friction and so forth) retarded by the restraining action of the string? Let FA on the straight line FAD be the direction in which the moving weight A has arrived at the point A, on the tangential straight



line FAD. Now if the angle FAC were greater than a right angle in any degree (e. g. fAC), it must be admitted that the string could not then retard the motion of A in the direction fAd. If on the contrary the angle DAC were greater than a right angle in any degree (e.g.) 'dAC, it must be admitted that the string would then retard the motion of A in the direction 'fA'd. But the tangent is necessarily at every point in the circle at right angles to the radius; *where* then can any retarding effect commence? *in what* does it consist? or *what influence* is there to which it can be attributed? If we can suppose the radius AC, or the string represented by it, to be of infinite length; then there would be no restraint, and A would move in an absolutely straight line. Or if we suppose the radius AC to be of indefinitely great length; the deviation of A's motion from a straight line through any given space will be indefinitely small, nevertheless every point and every part in the line will represent a point and a part in the arc of a circle, and if there be no retarding force at any one point or any one part, neither can there be at any other point or part, because that would suppose a dissimilarity in the motions or relative positions of the parts contrary to the conditions of the case under consideration. It may be argued that (1) where there is no retardation, as in the supposed instance of an infinite radius, there is also no restraint; but that (2) when the motion is restrained and caused to deviate from the straight line into the arc there is retardation, and that the force causing or resulting from such retardation is that known as centrifugal force, and is represented or measured by the tension of the string. In order to examine this objection we will now take the subdivision of the case (b), and suppose an elastic spring to form a part of the connecting line by which the weight A is attached to the central point C, (Fig 2.) The weight A is assumed to start from the point A, at the same distance from the centre and to move with the same velocity

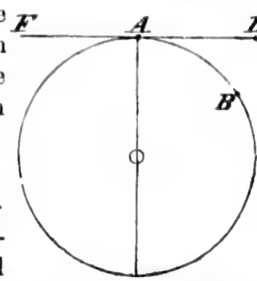
as before, which would cause it to arrive at D in the time (t); the effect will now be that the tendency of the weight A to move in the straight line will, in the first instance, be only partially restrained by the string, because the elastic spring, connected therewith, will be extended and allow the weight A to move outside the circle ABE. We will suppose this extension, which will be limited to a certain short time, sufficient to allow the weight A to arrive at the point l, and afterwards to arrive at the point H, when the elastic force of the spring reacting towards the centre C, may be supposed to become equal to the centrifugal force of the weight, and equality being established, the weight A will continue to move in the



greater circle of which the arc HK forms a part. Now, if we take any point l, at which A arrives when the elastic spring has been partially extended, then since the weight A has moved from the point A, in the curve Al, and A is a point on the circle ABE, and the curve Al is between that circle and the tangential straight line AD, the direction of the motion of the weight A at the point l will be the tangent to the curve Al, and which may be represented by the line mln. Join Cl, and through the point i, where Cl cuts the circle, draw the tangent o. p. Evidently the angle Cln is greater than a right angle (for the angle Cip is a right angle and Cln is greater than Cip,) and consequently it must be admitted that the string Cl retards the motion (*i. e.* diminishes the velocity) of the moving weight A. When the weight A has arrived at H, then since the centrifugal force is unable to extend the spring any further, and the reactionary force of the extended

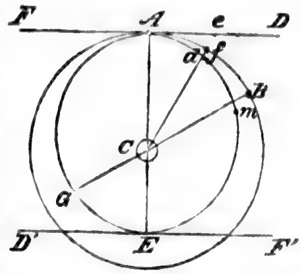
spring is unable to overcome the centrifugal force, the conditions of the case now become similar to those of the subdivision (a), with the difference, however, that the weight A will now revolve in the greater circle of which HK is an arc, and will move therein with a velocity less than the velocity which it had when leaving the point A (*i e.* less than the velocity which would enable it to travel from A to D in the time (t). But the spring which forms part of the line connecting the weight A with the central point C is now extended; and the force employed in extending the spring is equivalent to the loss in velocity of the revolving weight A.

Case (2). Under the conditions of this case, the centrifugal and centripetal forces being equal (*i. e.* the central gravitating force being counter-balanced by the tendency of the body A to move along the tangential straight line), the motion of the body A will be restrained from deviating out of the circular orbit of revolution, and the velocity of revolution will be neither accelerated nor retarded. The body A (Fig 3) will arrive at the point B on the circle in the same time it would otherwise have arrived at the point D on the straight line, therefore AB = AD.



Case (3). The conditions belonging to this case will be included if we suppose that the central gravitating force is in the first place more than sufficient to restrain the moving body from increasing the distance between itself and the central body; the superior gravitating force therefore causing the revolving body A to deviate from the circle and to approach the central body.

The body A (Fig 4) is moving at the point A, in the direction FAD, with the same velocity as before, and would arrive at D in the time (t). By case (2), if the gravitating force acting towards the centre was equal to



the centrifugal force, the body A would move in the arc A fB of a circle; retaining always the same distance from C as it had when leaving the point A; but the gravitating force is now greater than equal to the centrifugal force, and consequently A, when it has moved through half the time $(\frac{t}{2})$ has been caused by the superior gravitating force to approach C, by the space contained between the point f on the circle and the point on the curve Adm, at which the line fC cuts that curve (as this point would be very near to d, we may denote the space for illustration as fd.) The space fd will therefore represent additional motion imparted to the body A, and which, being compounded with the motion in the circle, will cause A to move with increased velocity, and to arrive at the point d in the same time $(\frac{t}{2})$ in which it would have arrived at e, on the line FAD, (Ae being the half of AD, and Ad being greater than Ae), but in moving through the remaining half of the time $(\frac{t}{2})$ the body A is caused to continually approach the central body, and at the end of the time (t) A arrives at m; the distance Am on the curve being greater than the distance AB on the circle which is equal to the distance AD on the line FAD; and the distance dm, is greater than the distance Ad, because the motion is thus far continually accelerated, (*i. e.* so long as the moving body actually approaches the centre of gravitation.) The difference by which the space Am on the curve, is greater than the space AB on the circle, measures the additional motion which has been imparted to the moving body by the superior gravitating force, and it represents the quantity of motion (or space) by which the body A has approached the centre. When the body A arrives at the point E

opposite to A (at the opposite extremity of the major axis of the ellipse) A has approached the central body by the difference between the distances AC and EC; and if A were supposed to be at rest (deprived of angular motion) at each of these distances from C, viz., AC and EC, then would the gravitating influence be inversely proportionate to the greater and the lesser distance viz., inversely as AC : EC. * But at the point E the angular velocity of the body A moving in the direction FED is considerably greater than at the opposite point A; and the areal velocity is also greater, *i. e.* the distance to which A would move from E in the direction FED in the time (t) would be greater than AD. Since the gravitating influence is uniform at any given distance, and since it is continuous, the amount of force exerted being directly proportional to the time during which it is exerted, it is evident that in the case of a body passing by or revolving around a second body, the gravitating influence or attractive force will be dependent upon the angular velocity of the moving body, and inversely proportional to that velocity. Therefore at the point E, the velocity of A's motion having increased in more than the inverse proportion of AC to EC, * the attractive force (effective gravitating influence) exerted upon A, is so much less at the point E, than at the opposite point A. The relative proportions of the gravitating and centrifugal forces are therefore reversed, and consequently the gravitating influence is now insufficient to restrain the body A, from increasing the distance between itself and the central body. When A, has moved from E, through an angular distance equal to that of ACB, the point G at which it will then have arrived will be at a distance from C greater than CE. The increase in the distance, viz., the difference between CE and CG will be in effect similar to the increase in the

* Note. The reason for thus stating the proportion will be presently explained, the Newtonian theory of gravitation would require as $AC^2 : EC^2$.

length of the connecting line by extension of the elastic spring (in sub-division b, of Case 1) and the effect will be to retard the motion of the body A. The diminished velocity of A at the point G will, however, be still proportionately greater than the gravitating influence, and consequently the body A will continually recede from C, and the velocity of its motion in the elliptical orbit will continually decrease, until the point A at the opposite extremity is again arrived at when the former conditions will be restored.

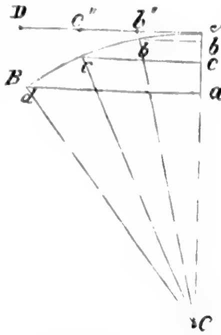
OBSERVATION.—The momentum of the body A in its approaching and receding motions relatively to the centre, *i. e.* its tendency when approaching to continue to approach, and when receding to continue to recede, will be hereafter alluded to; its effect is to exaggerate (to increase) the deviation from the circle, causing a greater eccentricity in the orbital revolution; and to give permanency to the deviation by preventing the restoration of equality of force and uniformity of motion.

The principle of equal areas described (passed over) by the radius vector of a moving body revolving round a central body and retained in its curvilinear orbit by the influence of gravitation (demonstrated by Newton in the first propositions of the "Principia,") may be shown to be the consequent or resultant of the relationship between the two forces, the centrifugal and the gravitating forces, and of their compounded action on the moving body.

NOTE.—*Newton's demonstration is here provisionally accepted as sound; but that demonstration includes and is indirectly based upon an assumption that the areas of circles vary as the radii (or as the perimeters) of the circles, i. e., in simple proportion; Now, this is not true in fact, because the areas of circles vary as the squares of the radii; consequently the supposed demonstration must be unsound. This case will be examined later.* *(See page)*.

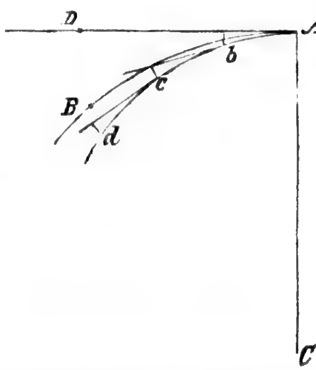
In the usual explanations and illustrations of Newton's demonstration of this principle it seems as if the nature of

gravitating force, viz., as being compounded of gravitating influence and time, has not been correctly understood; for example (Fig 5):—a body A is supposed to revolve around a central body C, being retained in its orbit by the influence of gravitation acting from the central point C (Fig 5). We suppose the successive curvilinear spaces Ab, bc, and cd to represent the spaces successively passed through by the body A, in its orbital motion, each one of them in the same equal definite time (t). Now the explanation usually given of such a case is to the following effect: it is at once assumed, notwithstanding that the body is moving in the direction of the tangent, that the central gravitating influence acting upon

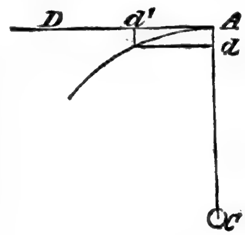


the moving body at the point A, takes effect by causing motion in the direction AC, and A is stated (or supposed) to commence falling towards the central body; and as the orbital motion proceeds, to continue falling with accelerated speed according to the law of gravitation applied to falling bodies. The analysis is sometimes shewn by drawing a line from that point arrived at by the moving body in the time (t) in the circle or orbit of revolution, (*i. e.* by drawing the sine of the arc) at right angles to AC, and then taking the distance of the point where the line so drawn cuts the radius AC from the point A; that distance is then stated to be the space through which the moving body has fallen towards the centre whilst moving through that portion of its orbit, thus Ad', on the line AC, would be considered the space through which the moving body A had fallen towards the centre C in moving through the arc or curvilinear fraction of the orbit Ad. Similarly Ab', Ac', would be considered as spaces fallen through by the moving body in travelling from A to b, and A to c

respectively, in the orbit. An apparently similar, but not equivalent, mode of explanation is by drawing the tangential line AD and deriving the space moved through in the orbit from the assumed motion in the direction AD by drawing a perpendicular from the assumed point of arrival on that line AD to the arc or curve representing the orbit, the perpendicular is then considered to be the space fallen through towards the centre of gravitation, and the motion in the orbit or arc is supposed to be compounded of the perpendicular motion towards the centre and the horizontal motion in the direction of the tangent. Another way in which the same (last) explanation is varied, analyses the composition of the orbital motion by subdivision of the curve into shorter spaces; instead of drawing the one tangential line AD from A, tangents representing equal divisional periods of time are drawn one to each of the fractional curves which constitute the divisional spaces of the orbit. Thus (Fig. 5 a), if Ad represent the whole time (t); then Ab=$\frac{t}{3}$; Ac=$\frac{2t}{3}$; Ad=$\frac{3t}{3}$=(t). From the terminal points of each of these divisional curves, the tangent is drawn representing the space through which the body would pass in the next equal division of time; then from the terminal point of this tangent the perpendicular is drawn cutting the curve of the ellipse; and from which point again the next tangent is drawn. In fig. 5 (a) the perpendiculars b, c, d, would be considered the falling motions belonging to the curves, and this perpendicular motion compounded with the tangential is supposed to produce the curvilinear motion through that space. Hence the perpendicular c is greater than b; and



d is greater than c ; also the curve b c, is greate than Ab ; and cd greater than b c. The erroneous assumption on which these explanations are based leads (1) to a false conclusion as to the quantity of motion imparted to the moving body by the gravitating force, and (2) to an unsound inference as to the composition or nature of the gravitating force itself. In fig 6, the case illustrated is that of a body moving around a central body with a velocity such that the centrifugal force equals the attractive force, and consequently the moving body can neither approach nor recede from the central body. The case is essentially similar to that of Case 1 (a), where the moving body was attached to the central point by an inelastic string or link, and in which it was shown that the velocity of the moving body was neither increased nor diminished by the guiding or restraining effect of the string. (Note. It has been explained in a previous note that an oscillating effect of approximation and recession takes place in the case of a body revolving around a centre of gravitation as above. The connection with the central point is here considered as equivalent to a rigid inelastic medium, whereas the radius-vector in this case (that of gravitation) may be correctly considered as extremely elastic ; this difference, however, does not in the least affect the argument here put forward ; and it will, for the moment, suffice to observe that this elasticity is also discarded by those teachers, the correctness of whose theories and conclusions we are here disputing.) The illustrations and explanations given in the Reference from the works of Whewell and Lardner show that in the case of the moving body revolving in a circle, the erroneous assumption leads immediately to the conclusion that the velocity of the moving body is increased



by the gravitating force, but which is contrary to fact, and it is indeed manifestly impossible that any acceleration of motion can take place in the manner so supposed. The unsound inference as to the nature of the gravitating force may be illustrated by Fig. 5(a) (page 21) which belongs to a case similar to that previously investigated in Case (3), viz., that wherein the centrifugal force of the moving body is less than the gravitating force, and in which consequently the moving body is made during a part of its revolution to approach the central body, and it was shown that in such a case the velocity of the moving body is increased. In Fig. 5(a), for example, the moving body will travel through the distance Ad of the curve, in the same time (t) in which it would have moved from A to B in the circle, and Ad is greater than AB. This acceleration arises from the actual additional motion by which the moving body approaches the centre, and which is compounded with the motion in the direction of the circle. The spaces Ab, bc, cd, in the orbital curve represent spaces successively passed through by the moving body in equal increments of time, but the spaces are unequal because, since the motion is constantly accelerated from the point A, each successive space is greater than that preceding it. Now if we proceed by the usual method to investigate the gravitating force to which the moving body has been subjected whilst moving through these successive spaces, it will be made to appear that the amount of gravitating force exerted is greater in each successive space in the same proportion that the space itself is greater than the space preceding it; for instance, in the same Fig. 5(a): if a tangent to the curve b c, be drawn from the extremity b, and a tangent to the curve c d, be drawn from the extremity c; and a perpendicular to each of the tangents be drawn, then according to usual teaching these perpendiculars represent the effective gravitating force exerted throughout each space respectively, and the perpen-

dicular d is greater than c, and c greater than b, in the same proportion that the space cd is greater than the space bc, and the space bc greater than the space Ab. But the gravitating force is the resultant of the gravitating influence and the time during which the influence is exerted; and the time represented by each of these spaces is the same. It is true that since the distance of the moving body from the central body is slightly lessened, the intensity of the gravitating influence is increased proportionately to the space by which this distance is diminished, but the increase thus arising is extremely small in amount and not nearly equivalent to the difference inferred from the erroneous assumption in question*. The unsound inference as to the gravitating force, and therefore as to the composition of the motion, arises out of the assumption that the revolving body moves tangentially to the curve. In the instance last given. (Fig. 5a) the curve Ab and the curve bc, would be, if the velocity of the moving body were such that the centrifugal and gravitating forces were equal, the equal arcs of a circle, and of which arcs the perpendiculars b, c, and d, would be equal. Now if an additional impulse were to be given to the moving body at the commencement of the second curve, and the velocity be thereby increased, then would the second be greater than the first arc, and the perpendicular of the second proportionally greater than the perpendicular of the first; the gravitating force exerted during the progress of the moving body through each of the arcs respectively would have been the same, because the time was the same, and the distance of the moving body from the centre of gravitation was through-

* Note This actual increase in intensity, small as it is in amount ought not to be taken as a deduction from the error, since it is, properly considered, quite distinct. If the revolution is in a circular orbit, or even if the deviation is to a curve outside the circle when the intensity would in fact decrease, the same erroneous method would still lead to the supposition of an increasing gravitating force for each successive division of equal time.

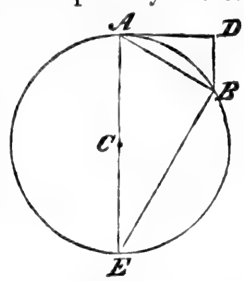
out the entire distance the same. Nevertheless the perpendicular of the second would be greater than that of the first, and this difference would, by the usual method of illustration, be made to demonstrate a greater amount of gravitating force exerted. The correct explanation may be thus stated. During the progress of the moving body through the first curve its velocity has been increased by the additional motion derived from the approach of the body towards the centre, the quantity of which motion is measured by the deviation of the curve from the arc of a circle; the length of the curve is dependent upon the velocity, because the curve represents the space moved over in the constant and definite time, and the length of the curve throughout must have been greater than the length of the arc would have been. The second curve commences with the increased velocity with which the first terminated, and if the distance of the moving body from the centre remained the same throughout the second curve, that curve would be an arc of a length dependent upon the initial velocity, which velocity would in that case remain the same throughout the curve; but the gravitating influence is supposed to be still superior to the centrifugal tendency, and consequently an additional motion of the moving body towards the centre is again compounded with the motion of the body in the orbital curve of revolution, and a further increase of velocity and a further deviation of the orbital path from the arc of a circle to the elliptical curve is the consequence; as before, the deviation measures the motion towards the centre, and the increased length of the curve exhibits the increased velocity, or as it may be called the accelerated compound motion of orbital revolution.

REFERENCES.

Whewell's Mechanics, page 159 (Centrifugal force.)

(167). "If a body is made to describe a circle with a uniform velocity it must be acted upon by a force tending towards the centre of the circle," &c., &c.

168. "Prop. When a body describes a circle with a uniform velocity, and is retained in its path by a force tending to the centre, this force is represented by the square of the velocity divided by the radius.—Let v be the velocity, r the radius, f the force which acts towards the centre. Let t be the small time in which the body, not acted upon by the central force, would describe the small portion AD of the tangent; and let DB be the deflection by which the body is brought to B.



Hence at the limit

$$AD = vt. \; BD = \tfrac{1}{2} ft^2 .$$

But if AE be the diameter, the triangles EAB, ABD are similar; for the angles ADB, ABE are right angles, and EAB, ABD are equal.

Hence EA : AB :: AB : BD; . . . . therefore EA × BD = AB × AB; . . . . . . and at the limit

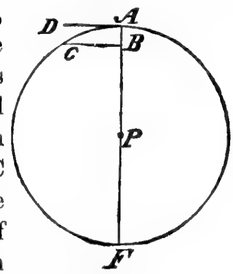
$BD = \frac{1}{2}ft^2$, ... $AB = v\ t$; . . . . hence

$$2r \times \frac{1}{2}ft^2 = (vt)^2 \quad . . . \text{Therefore } f = \frac{v^2}{r}$$

This explanation (or demonstration) contains the remarkable substitution (as above) of AB for AD, an assumption apparently that AB and AD are equal. Since it is obvious on mere inspection that AB is greater than AD, the idea suggests itself that this substitution must be an oversight or misprint; but on examination it appears to be an error of a different kind, viz., the statement as an axiom or fact of an unsupported assumption which is not manifest and is apparently unreasonable. It

appears that the statement has been made under an impression that AD compounded with DB results in AB, (viz., that A arrives at B in the same time it would if not compounded with DB, have arrived at D). But not even a reference to such a supposition (or demonstration) could justify in this place the substitution of AB for AD, since here the object of the reasoning and the illustration is particularly to ascertain and demonstrate the value of AB, and its relation to that of AD, and of other quantities. (Observation. To avoid misunderstanding it may be well to observe that the above calculation would be in itself correct, if AB, a part of the circle, were taken in the first instance as the space moved through in the definite time (denoted by t). But AD, representing the tangential motion from the point A must be made equal to AB, the arc; and consequently DB will not be a (perpendicular) straight line but a curve, and will be greater than DB shown in the figure.)

To show that this unsupported assumption is not confined to Whewell's treatise but is at the present time a part of the recognised scientific teaching on the subject, we refer to Lardner's Mechanical Philosophy, page 147, (fig. 63): "Let P be the fixed point to which the string is attached. Let A be the ball, and let ACF be the circle in which the ball is whirled round. Let AC be the small arc of this circle moved over in a given interval of time. Starting from A the motion of the ball has the direction of the tangent AD to the circle, and it would move from A to D in the given interval of time, if it were not deflected from the rectilinear course; but it is deflected into the diagonal AC, and this diagonal by the composition of forces is equivalent to two forces represented by the sides AD, AB. But the motion AD, is that which the body would have in virtue

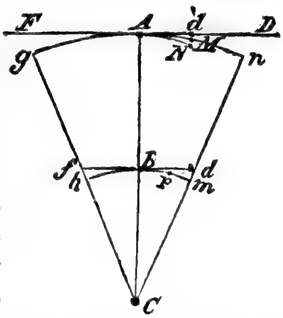


of its inertia, and therefore the force AB directed towards the point P is that which is impressed upon it by the tension of the string, and which, combined with the motion AD causes it to move in the diagonal AC." Here we find the same unsupported assumption in a somewhat different form. The arc AC is taken as representing the space moved over in a given interval of time by A, which is attached by a string to the central point P; it is then stated that if the motion of A were not deflected by the attachment to the central point, A would move (i. e. would have moved) in the same interval of time to D. But AD is less than AC, therefore the velocity (in the deflected movement to C) has been increased, and this increase in the velocity is distinctly attributed to the tension of the string causing a motion in the direction of the centre P, and this motion represented by AB is assumed to compound itself with the motion AD and to result in AC. If it be granted for a moment that such assumption may be true, it would immediately follow that the case must be one of uniformly accelerated motion increasing from A throughout every divisional part of AC, and the velocity of A, when it arrives at C, must be accordingly greater than when it passed the point at A, and so continue to increase throughout the circle (i. e. throughout every part of each successive revolution). This obvious corollary is apparently quite overlooked. (Observation. To substantiate the correctness of that teaching now recognised as sound, or in other words to demonstrate the conclusion thus arrived at by Whewell and Lardner, it would be necessary to adopt as a postulate, or to demonstrate in the first instance, that if a force act continously on a moving body at right angles to the direction in which the body is moving, such force, by compounding itself or its effect with the motion of the body, produces accelerated motion in the body. (increased velocity.) Where is demonstration on this point to be found? Has any one even ventured directly and positively to assert such a proposition?

THE LAW OF GRAVITATION.

The Newtonian theory of gravitation assumes that the intensity of the attractive force (gravitating influence) varies inversely as the square of the distance, increasing as the square of the distance decreases, and vice versa. Is such assumption supported by the facts?

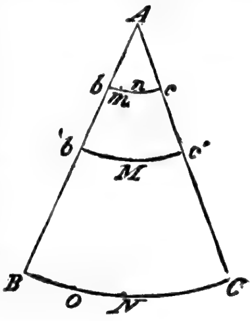
(Fig. 7.) Let C be the centre of gravitation, B a body moving in the direction fBd at the distance CB from C, and let the velocity (v) be such as would carry B in the time (t) to d, at the distance Bd, and let the gravitating force be equal to the centrifugal force; B will therefore move in the arc of a circle, and will arrive at m in the same time it would have taken to arrive at d in the tangential direction fBd. Now let us suppose the intensity of the gravitating influence acting from the centre reduced to



one half; and the velocity with which B moves from the point B, in the direction Bd, also reduced to one half: it is evident, since the centrifugal force (the mass and density of the body and the radial distance remaining the same) is directly dependent upon and proportionate to the velocity, that by reducing the velocity, the centrifugal force is also reduced to the one-half; but previously the centrifugal force and the gravitating force were equal, therefore they must be still equal, because the half of the one must be equal to the half of the other, and in the same time (t) the body B will have arrived at a point p in the arc of the circle at half the distance of the point m from B in the same circle. We will next consider the nature of the centrifugal force. Since if the body B was allowed to move straight fore-

ward in the direction of the tangent Bd there would be no centrifugal force, it is evident that the centrifugal force is in the first place a consequent of the compelled deviation from the straight line into the arc of the circle, caused by the restraining or governing influence acting upon B from the centre. And again, since in every point or part of the circle the tangent is at right angles to the radius, and the relationship of the tangent to the arc which it touches is always the same, it is evident that the amount of the centrifugal force is also dependent upon and directly proportionate to the velocity or speed of the moving body; for any definite amount of space moved through by the body in a given time is productive of a definite amount of centrifugal force, and if the velocity be doubled, then double the space is moved through in the same time; if quadrupled, then four times the space is moved through in the same time, and the amount of centrifugal force developed is necessarily doubled or quadrupled accordingly.

In order the more readily to appreciate the effects consequent upon variations in the proportions of the opposing forces respectively, and in the distance at which their influence acts upon the moving body, it is desirable to examine briefly 'the relation of the enlarged circle to the lengthened radius.' (Fig. 8.) With centre A and radius Ab describe the arc bc; double the length of the radius through b, and with radius A'b, describe the arc 'b c'; double the length of the radius through 'b, and with radius AB describe the arc BC. We have now three arcs subtending the same angle, and consequently similar arcs, that is to say similar fractions of the circles to which they respectively belong; and the three arcs are related to each other in such



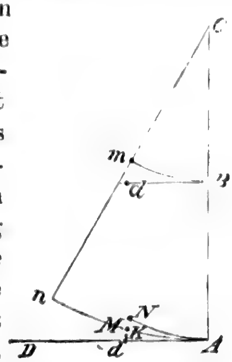
wise that one half the arc 'b 'c, (i. e. 'b M or M c') is equal in length to the first arc bc; and again one half the arc BC (i. e. BN or NC) is equal in length to the arc bc, and also one fourth the arc BC (i. e. BO or ON) is equal in length to the first arc bc. And further it should be observed that since the arcs are similar fractions of their respective circles, that they therefore contain equal amounts of curvature, and since the first arc is half the length of the second arc, the first arc contains proportionately twice the amount of curvature; and again, since the second arc is half the length of the third arc, the second contains proportionally double the curvature contained by the third; and the first arc contains four times the curvature contained in the third arc. Hence if one half the second arc as 'bM were to be increased to double the length without increasing the curvature, it would become converted into an arc belonging to the larger circle of which the third arc BC is a fraction; and the half arc 'b M so increased to double the length would be then similar and equal to half of the arc BC.

Returning to Fig 7. we will now suppose the moving body to be removed to twice the distance from the centre, and let CA be the radius-vector or distance so duplicated. If with the distance CA we describe an arc An, intercepted at n, by a radial line drawn through the terminal point of the arc Bm, the arc An so described is necessarily twice the length of Bm, and contains the same amount of curvature, and consequently, if we bisect the arc An we then have an arc AM equal in length to the arc Bm, and containing half the curvature comtained in the arc Bm. Since the moving body is now twice the distance from the centre of gravitation, the restraining force, on the supposition that the intensity of the influence decreases in some inverse proportion to the distance, will be now less. What proportion does the decrease in the intensity of the gravitating influence bear to the increase

of the distance? (if, taking the place of the moving body at A with a radius-vector or distance the same as at first, that is with the distance BA (because BA is equal to CB) we describe the arc AN equal and similar to the arc BM; then, if we suppose the intensity of the gravitating influence to remain as before, the point N is the point at which the moving body would arrive in the time (t) and dN (equal to dm) would be the deviation from the tangential (straight forward) direction of motion.) Let us suppose the decrease of intensity in the gravitating influence to be in simple inverse proportion to the distance, that is to decrease just so much as the distance increases; then, since the distance has been doubled, the intensity of the influence will be now the one half of that to which the moving body was subjected at its former place, viz., at B. The velocity of the moving body is (by the supposition) the same as before, and it will consequently in the same time (t) move to a distance from A equal to the distance to which it previously moved from B (because there is no obstacle in either case to retard the motion). By the supposition the intensity of the gravitating influence is the one-half and the space moved through in the same time is as before: evidently therefore the deviation will be now the one half of what the deviation was before, and the curve moved through will still be the arc of a circle, because an arc described with the distance twice as great will give the required deviation, and the point at which the moving body will now arrive will be the terminal point M, of the arc AM, equal in length to the arc Bm, but containing only one half the curvature contained in that arc; that is, only half the deviation from the tangential line. ($\frac{dm}{2} = dM$.)

But the assumption of Newton's Theory of Gravitation is that the intensity of the gravitating influence (attractive force) varies inversely as the square of the distance; that

is,—the distance being doubled, the intensity of the influence will be reduced to one-fourth. Now, if we test this assumption by considering what, if it were true, would actually take place under such conditions as those just now supposed, it becomes at once apparent that the moving body could no longer move in the arc of a circle, because the intensity of the influence being only the one-fourth, and the space moved through in the same time being the same as before, the deviation from the tangential line (direct line of motion) will be only the one fourth as great as before, consequently the point at which the moving body arrives will be the point K (Fig. 9),* half the distance between the point d, and the point M; and the curve AK, very nearly equal† in length to AM, would be the curve representing the path of the moving body; on the assumption that the intensity of the force decreases inversely as the square of the distance. It follows that, if such assumption were true, the moving body would continuously recede to an indefinite distance from the centre of gravitation. (The orbit of revolution would become a spiral or helix with the curve increasing outwards.)



What are the facts which are supposed to support the assumption? Taking for example the case of the earth in its revolution round the sun; it is known that the orbit in which the earth revolves is elliptical, and therefore when the earth is in that part of its orbit at the least distance from the sun,....that is, at one extremity of the major axis of the ellipse called the perihelion,....we have an instance wherein

* Fig. 9 repeats Fig. 7.

† *Very nearly*, because the motion will now be very slightly retarded, as the angle contained by the tangent to the curve, and the radius, will be greater than a right angle. See case 1 (b).

the intensity of gravitating influence is unable to neutralise the centrifugal force; or, in other words, wherein the proportion of the gravitating force to the velocity is insufficient to restrain the moving body from increasing the distance between itself and the centre of gravitation. At the moment of perihelion the earth is moving at right angles to the radius-vector. As the motion proceeds, the distance of the earth from the sun, in consequence of the superior velocity in this part of the orbit, increases; that is, the deviation from the tangential direction of motion becomes less than that required by the arc of the circle. Now if we assume that the decrease of intensity of the gravitating influence is in simple inverse proportion to the distance (i. e. varies inversely as the distance), it will be apparent that when the recession of the earth has increased the distance to a certain limited extent from the centre of gravitation, the gravitating influence will become sufficient (or more than sufficient) in proportion to the velocity, to restrain the earth from receding to a greater distance. The distance of the earth from the sun at which the centrifugal force is in equality with the attractive force must evidently be the actual average distance from the sun. If this distance is correctly ascertained and also the distance at perihelion (i. e. the least distance) we have then, since the time of the entire revolution and the angular velocity through definite fractions of the orbital revolution are known, the means of determining the velocity at the perihelion in excess of the velocity required to equalize the attractive force acting from the centre of gravitation.

Taking the average distance of the earth from the sun at	95,000,000	miles.
The distance at aphelion	96,595,000	,,
" " perihelion	93,405,000	,,
	3,190,000	,,
Gives the motion in 91¼ days at the greater distance (i.e. one-fourth the orbital revolution)	151,731,426	,,
The same angular motion at the lesser distance requires an areal velocity of	146,720,574	,,
The difference	5,010,852	,,

gives the apparent increase in angular velocity at the lesser distance.

Supposing the actual or areal velocity to remain the same as at the greater distance, then .... $\frac{146,720,574,}{5,010,852,}$ gives $\frac{1}{29\frac{1}{2}}$ as about the apparent increase in the angular velocity.

By the approach of the earth and consequent deviation of the orbital path from a circle to an elliptical curve, there will be an actual increase in the areal velocity. This will be the ultimate velocity obtained by compounding the motion towards the sun with the motion in the circular orbit at the greater distance.

Taking the time of the approach as 91¼ days:—Then
$\sqrt{(151,731,426^2+3,190,000^2)}-151,731,426=151,764,956-151,731,426$
$=33,530,$ . . . . and $33530 \times 2=$
* 67060 miles in 91¼ days,
(i.e. an increase of about 1 mile in 2260 miles.)

The earth's increased velocity now causes the centrifugal force to exceed the gravitating influence, and consequently the earth commences to recede from the sun. We will suppose this recession to have continued until equal to the one sixtieth part of the radial distance measured at the place of least distance from the sun;—what will be now the conditions of the earth's orbital motion? By the Newtonian theory the attractive force will have decreased in an inverse proportion to the square of the increased distance; putting d for the distance at perihelion, and g for the gravitating influence; then, g at increased distance : g at perihelion $:: d : d+\frac{d^2}{60}$
$901,313,190,562,500-872,449,402,500,000=28,863,788,062,500$;
the decrease in the attractive force being therefore very nearly one-thirteth. But the increase in the distance is only one sixtieth, consequently the centrifugal force will be now still greater proportionately to the gravitating influence than it was at the lesser distance, and therefore the earth must continue to recede from the sun indefinitely.

If, on the contrary, we assume the decrease in the attractive force to vary inversely as the distance simply, then

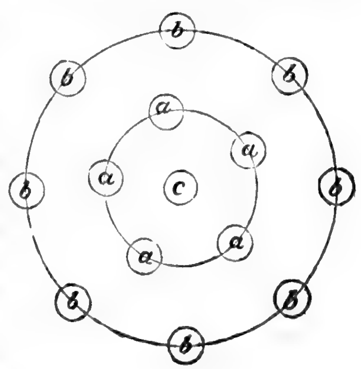
* The final velocity by the law of uniformly accelerated motion.

as the earth recedes the alterations in the conditions which were caused by her approaching the sun are simply reversed, viz., a considerable apparent decrease in the angular velocity, and a comparatively small decrease in the areal or actual velocity takes place. This decrease both in the angular and in the areal velocity is precisely equal to the increase during the motion toward the sun; the former by causing a proportionate decrease in the centrifugal force counterbalances the equal decrease in the intensity of the gravitating influence (attractive force), and the latter is the immediate cause which determines and regulates the average distance of the earth (or other planet) from the sun.

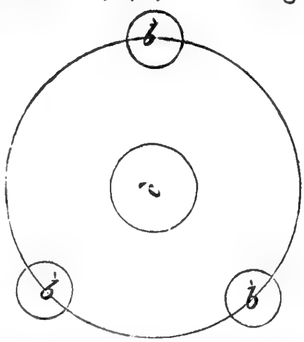
The motion toward and recession from the sun may be compared to the vibrations of a pendulum, the alliance between these manifestations of gravitating force being in fact very close. The amount of these vibrations (oscillations) causing the greater or lesser deviation from the circle known as the eccentricity of the orbit, is, however, partly dependent upon the disturbance (or effect) arising from the gravitating influence of other bodies.

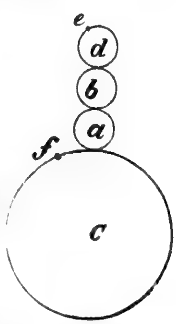
GRAVITY.

Let a number of bodies a, a, a, &c., (Fig. 10) revolve in the same circle (i.e. at the same definite distance and in in the same plane) around a central body, and let a number of bodies b, b, b, &c., also revolve in another circle at twic the distance from the central body, and let the intensity of the gravitating influence on the surface of each of the bodies



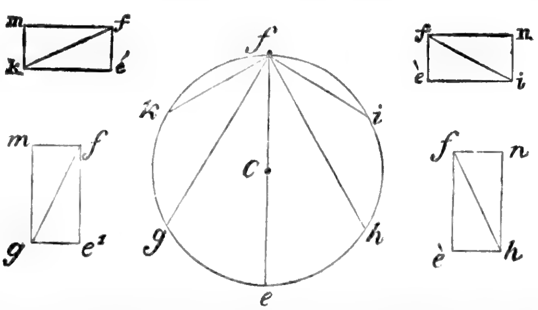
a, a, a, . . . and also on the surface of each of the bodies b, b, b, . . . and on the surface of C, be the same as on the surface of the earth. (or other definite amount of intensity may be supposed, so that it be the same in all the bodies). Now supposing the bodies contained in the inner circle, viz., a, a, a, . . . brought together and combined with the central body C into one body, as in Fig. 11, and also the bodies contained in the outer circle b, b, b, . . . brought together and combined into only three bodies: what alteration, if any would such re-arrangement of these bodies occasion in respect to the intensity of the gravitating influence on the surface of the remaining bodies b, b, b and C? Since the mass or bulk of these compounded bodies would be now so much greater than formerly; would the intensity of the gravitating influence on the surface of each, be therefore necessarily increased? And if so, would the increase in gravitating influence be equal to the increase in the mass or bulk? To take the case in the first instance in a simpler form let the several bodies a, b, d, (Fig. 12), be piled upon the lower body C, which is formed of a number of bodies similar to a, b, d, united into one. It is evident that the body a, next to C will be subjected to the gravity of the bodies b, and d, above it, and will be under pressure; and also that the part of a, next to C will be under the gravity and pressure of the upper parts of itself as well as of the other bodies above it; and that similarly the lower part of b, will have to sustain the





weight of the upper parts of itself as well as of d; and so also of the uppermost body d. Now let us suppose the small body e, upon the upper surface of d, and another small body f, of the same size and equal to e, upon the surface of the great body C; which of these bodies, e or f, will be subject to the more intense influence of gravitation? It might be argued that the gravitating influence of a, would be added to that of C. and that the influence of b, and of d, would be further additions all combining and causing a greater effect at the upper surface of d, and thereby subjecting the small body e, to more powerful attractive force (influence) than that acting on f, at the surface of C. Such argument would certainly not be supported by the facts. Assuming the attractive force to vary inversely as the distance from the centre of force, it follows that if the distance of the upper surface of d (i.e. the length of the diametres of a, b, d combined together) is equal to the diameter of C; the combined influence of a, b and d only would require to be equal to two-thirds of that of the great body C in order that the effect on e, at the surface of d, should be as great as on the surface of C itself, but this cannot be; because the body C is compounded of bodies similar to a, b and d, and the influence upon a body [as f] at the surface of C, is the resultant of force derived from all the parts or bodies compounding C. It is convenient and for some purposes not incorrect to refer the forces to a central point, but in fact the force belongs to and emanates from every part and point of the sphere, and the action on the body at the surface is not only in the vertical direction but also in each and every angular direction in which a line can be drawn from any part or place in the sphere to the body on its surface. The body on the surface is not, however, acted upon with an equal amount of attractive force from every part or point in the sphere, but the amount exerted by parts of equal mass is dependant upon the situation of the part

relatively to the centre of the sphere and to the body on the surface; the actual proportionate amount of influence is measured by the angle contained between a line through the centre, joining the body, and a line joining the body with the part whence the influence is exerted. A little consideration will show that only those parts directly in the line from the body passing through the centre of the sphere can exert their entire influence on the body, that is, the whole of the direct influence belonging to their distance from the body; from all other parts of the sphere the influence proceeding from any part on one side of the central line, is in a greater or lesser measure opposed by an equal influence proceeding from the similarly situated part on the other side of the line; and this opposition will become greater and the acting influence less, directly in proportion as the angle contained between the line joining the body on the surface to the centre and the line joining the body to that point whence the influence proceeds becomes greater. This is illustrated at Fig. 13, where fe, represents the diameter or central line. Any part (every part) in the line fg. is op-



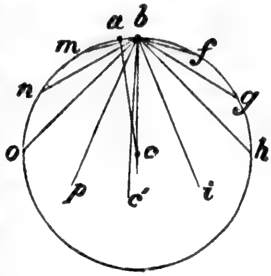
posed by a similar part in the line fh; of these mf, and nf, in the side figures represent the proportion of the influence exerted which is directly opposed and neutralized from the opposite side each by the other; and fè and fé the proportion of each which is directly effective on the body at f, the same as if that portion (of the force represented by

the line fg or the line fh) proceeded from a point situated on the central line. Similarly in the upper side figures; kf and if, representing the influence of parts and points situated in the lines kf, and if, of the centre figure, are resolved each of them into two forces exerted at right angles to each other, of which the one part directly opposes the similar part of the opposite force (as mf and nf and the other part acts directly in the vertical line as fè and fé. Returning to Fig. 12 it now becomes evident that the collective influence or attractive force of the three bodies a, b, d, together with that of C, acting at the distance Cd, at the upper surface of d, must be less than that of C itself acting at its own surface, because the distance is equal to the diameter of C, and C contains more than twice the gravitating mass contained by a, b, d, collectively. Considering the gravitating influence of all the matter contained in C, it may be correctly considered as acting collectively from the central point, and therefore the radius represents the distance at the surface (or the primary distance) and a removal to a distance equal to the length of the radius would reduce the intensity to one half; for example, taking the earth's diameter at 8000 miles, the intensity of the gravitating force at a distance of 4000 miles from the surface would be reduced to about the one half. (Note. It may be suggested, in the absence of any experimental or mathematical investigation, as being at least very probable that for a certain limited distance from the surface of the earth (or other planet) there will be but very little decrease, if any, of the attractive force at the surface. The reason of this will be at once understood upon reference to Fig. 13; because if the body f, were raised a short distance from the surface; the angles contained by the lines kf, gf, hf and if, respectively, together with the vertical line, would decrease and a larger proportion of gravitating influence thus become effective; for a certain short space this gain may equal the loss from the increased distance, and thus the decrease in attractive force for a few miles from the earth's surface

must be very small; perhaps there is no appreciable decrease within a certain distance). We will next suppose the three spherical bodies a,b,d, combined together with C into one spherical body $\bar{C}$; and, assuming the bulk of a, b, d, collectively equal to one third the bulk of C, we shall then get a sphere greater than C by one third. Now it is true that an increase of one third in bulk will be obtained with much less than one third increase in the length of the radius (or distance from the centre) but as this addition to the bulk is an enlargement of the lesser sphere C from the exterior, it is evident that only a small part of the additional matter will be so located as to act with direct influence on a body at the surface, and much of the matter so added will have a large proportion of its collective attractive force neutralised by the opposing forces of portions distributed over the various parts of the surface (*i. e.* the superficial parts of the sphere). Let us now consider whether a definite mass of matter undergoes any alteration, increase or decrease in intensity of gravity or attractive force, by an addition to its bulk of another mass of the same form of matter in the same state or condition: as for example, iron in the solid state. If a mass of iron be compounded with another mass of iron; is the compounded mass possessed of a greater degree of gravitating influence (attractive force) than was possessed by each of the compounding masses? In other words; is there in such a case an increase of intensity in the gravitating influence of the matter? It is unquestionable that in a collective sense the greater mass will have a greater amount of gravity, in proportion to its greater mass; because all the parts possess independently their proportion of influence (or force). The greater mass is therefore heavier; it has greater weight (the words heaviness and weight expressing this collective sense); but is the specific intensity of attractive force increased by or in consequence of the increased aggregation of matter? What are the known facts? A definite mass of iron weighs ten pounds. Another

definite mass of twice the bulk weighs twenty pounds. A third mass ten times the bulk of the first weighs one hundred pounds. Now if the three masses are allowed to fall from a definite height to the earth (say from 500 feet for instance) will the second descend twice as fast as the first, and will the third fall with five times the velocity of the second and reach the ground in a tenth part of the time occupied by the first? or if not, will the velocity of descent in the case of the second and third respectively be greater than that of the first? and in what proportion? It has been long since established and it is well known that there will be no difference in the velocity of descent or time occupied in the fall from the same height. The ten pounds weight (or a one pound weight) will reach the ground in precisely the same time as the one hundred pounds weight (or as a one thousand pounds weight, and so on). Supposing two planets, each of the size of the earth, were brought sufficiently near to each other, they would both move with equal velocity towards each other until they came into contact; and if, instead of equal sizes, one of them was half the size of the other, then the lesser would move towards the greater with twice the velocity with which the greater approached the lesser. Now if we take two spheres of iron weighing ten tons each and place them a short distance apart; will they (rush together) move towards each other with equal velocities? Or if the one weighs twenty tons and the other ten tons will the less approach the greater? No,—in either case the two bodies will remain apart; and no tendency even, to fall towards (or approach) each other will be apparent. Let the size of the larger be very greatly increased and the size of the smaller decreased, do we then find any apparent tendency in the lesser to approach (fall towards) the greater; for example, let us take a mountain of considerable height and having one of its sides nearly perpendicular, and near to that side and from the same height (as that of the mountain) allow a piece of iron (or other such

body) to fall to the ground; will the piece of iron in falling approach in any degree the mountain? It is well known that it will approach the mountain. Let us then again carefully consider the case of the two iron spheres under the actual conditions and circumstances belonging to it. Fig. 14 represents the earth, in order to imagine the two spheres visible we will increase their bulk and take the larger at 2000 tons (a) and the lesser at 1000 tons (b).*



By drawing a few of the lines indicating the direction in which various parts of the mass compounding the earth exert their gravitating influence, it at once becomes apparent that the attractive force of the larger sphere is opposed from the directions f, g, h, i, &c., &c., and is aided from the direction m, n, o, p, &c.; it is true that these opposing forces from the earth's mass would of themselves neutralise each other and leave a certain amount of vertical attraction (apparently from the centre) as the resultant, but when an additional (outside) source or subject of gravitating influence becomes effective, the whole must be considered together and separately (*i. e.* the whole as compounded of its parts, in order to appreciate the actual effect upon the lesser sphere b; and it becomes evident that the resultant will be a very slight deviation in the line of the apparent attraction of the earth, a deviation which (by enormously exaggerating the effect for the sake of illustration) may be indicated by the line b 'c, forming the angle 'c b c, with the vertical central line of attraction. When it is considered that on the scale here shown the size of (a) would represent a number of the largest mountains on the earth's surface

* Note. In fact on the scale of the figure a sphere of 2000 tons would be a point so small as to be scarcely visible. (This has reference to a figure of 18 inches diameter.)

united into one sphere, it will be readily understood that if (a) be reduced to the size of any ordinarily very large object the effect which its gravitating influence could have upon another object of the same or less size than itself would be almost or quite inappreciable, and notwithstanding this the actual gravitating influence exerted by it may be proportionately just as great or even greater than that of the earth; and that if the earth's influence was removed these two spheres would approach each other (rush together) with a velocity respectively proportionate to their bulks; if of equal size they would approach each other from a proportionately small distance with the same velocity as that with which the two planets of equal size would approach; if the one was twice the size of the other, then their respective velocities would be as two to one. (Note. Proportional (or proportionally equal) distance; if the planets were of the earth's size, half a diameter would be 4000 miles; if the two spheres were one foot in diameter, half a diameter would be 6 inches; but *comparative density* is here left out of consideration.*)

* It may be considered a corollary to the above,....that for bodies of equal density, the distance to which the collective influence of the spherical mass extends, must be directly proportional to the diameter of the body. This corollary is accepted. The modifying effect of differences in density may be thus shown:—If A, and B, are two spherical bodies of equal densities, and the diameter of A, is twice that of B; then the attractive force of A,—at a distance from the surface of A, equal to one diameter of B,—will be equal in intensity to that of B, at the surface of B. Now if the attractive force of B, is taken at a distance of 1 semi-diameter from its surface (which will be one-half of the intensity at the surface), then to reduce the attractive force of A, to equality therewith, the distance from A's surface must be 3 diameters of B, (i. e. 1 + 2 diameters). But if B, is more dense than A, in such proportion that the collective influence of B, is equal (absolutely) to that of A; then the attractive force of B, at a distance of 1 semi-diameter from its surface will equal that of A, at the surface of A; because for the collective influence to be equal, as by the supposition; the attractive force at B's surface must be greater than at A's surface, proportionally to the greater density), and if the distance from B, is then increased by 1 diameter; to reduce the attractive force of

Are we now in a position to answer the question asked with regard to figures 10 and 11? The intensity of the gravitating influence would be increased: and the increase would be directly proportional to the increased radial distance from the centre to the surface. (i. e. The intensity varies as the diameter of the sphere.) Thus taking the lesser sphere at ten miles diameter, . . . and the greater, at twice the mass or bulk; . . . putting g, for the intensity at the surface of the lesser sphere; and G, for the intensity at the surface of the greater sphere . .

Then; If $g = 15$℔ on the sq. inch,

$$G = (1.26\ g) = 18.9$$

Or taking the greater at twice the diameter of the lesser.

$$G = 30 \text{℔}.$$

Or taking the diameter of the greater to that of the lesser as 3 : 2; then, $G = \frac{3g}{2} = 22\frac{1}{2}$℔.

[NOTE 1.—Fifteen lb. on the square inch is about the pressure of the atmosphere, or the weight of a column of mercury 30 inches in height, and having a sectional area of 1 sq. inch. This quantity is taken here merely as a convenient and simple measure; it may be readily converted into the velocity of a falling body.
(e. g., . . . let $g = 57{,}900$ feet in 1 minute; and taking the diameter of the greater sphere to that of the lesser as 3 : 2; $G = (2\,g - \frac{g}{2})$ = the same space fallen through in 45 seconds.]

[NOTE 2.—The relationship of the intensity, at the surface, to the superficial area and to the diameter of the sphere, may be thus understood. If the sphere be increased to twice the diameter without alteration of the

A, to equality therewith, a distance equivalent to 1 diameter of B only, from the surface of A, will be required.

To this note may be appended a suggestion—that it might, for many purposes, be an improvement to define a sphere as having *no form* (i. e. negative); then all forms would be deviations from the sphere (and, in such sense, positive.)

intensity, then twice the diameter gives 8 times increase of the mass (or solid contents), and 4 times increase of the superficial area (or convex surface); hence, for every square foot (or each square inch) the effective influence is doubled; that is, the intensity varies as the diameter.]

In considering the case of a large mass of aggregated matter such as a planet, density must not be left out of consideration: evidently for any particular form (variety) of matter, the intensity of the gravitating influence must be proportionate to the density, because taking a mass of any size, each part and point contained in that mass exerts its proportionate influence, and all those parts or points will continue to do the same if brought closer together; that is, if the size of the mass was reduced by compression of its bulk, which would cause it to become more dense. Now if three bodies or particles of matter are arranged together so that one of them is between the other two, the middle body will be subjected to a pressure, and if spherical the tendency would be to flatten it at the two sides in contact with the outer bodies, and if surrounded by a circle of bodies all of which were attracting each other, the effect would be to subject it to a pressure on all sides; and again if this circle of bodies was surrounded by a second circle of bodies which also exerted an attractive force on each other, then would the bodies composing the inside circle be subjected to a pressure between those of the outside circle and the body occupying the centre; and the pressure on the central body would be further increased, and so on. Therefore as an aggregated mass of matter becomes very great, so does the pressure on that part occupying the centre or situated near the centre become very great, and consequently those forms of matter capable of undergoing compression into a smaller bulk which occupy such a position will be rendered more dense. It is, however, at least very probable that the maximum density which any of the various forms of matter are capable of acquiring

through compression would be arrived at in the centre of a planet of comparatively very small size ; and, if this is assumed to be the case, the density caused by the compression of gravitation would be no greater in a very large than in a very small planet, since the larger quantity of dense matter would be simply proportionate to the greater mass of the large planet. Since therefore the increase in the bulk or mass of a planet does not involve an equal increase in gravitating influence* (attractive force) at the surface of the planet, let us next enquire what the effect of such an increase in the magnitude or mass of a planet revolving at any definite distance from the sun would be on the motion, or on the orbital distance from the sun, of that planet. Let the earth be the supposed planet, and let the earth be enlarged to twice its present size (i. e. twice the volume or mass), the present average density to be also the average density of the duplicated earth, what would be the effect on the motion and on the orbital distance of the earth? At first it might be thought that, as the collective gravitating influence had been doubled, the earth would therefore fall towards (approach) the sun ; but the earth is revolving round the sun with a certain velocity, and the explanation already given will suffice to show on careful consideration that no alteration in the distance of the earth from the sun would be necessitated, nor would there be any change in the velocity of the earth's orbital motion, because the centrifugal force is dependent on the mass and velocity taken together, just as the effective gravitating force is dependent upon them : and consequently the centrifugal force will be proportionately increased and will neutralise the increased gravitating force. But the earth's satellite, the moon, would be greatly affected, because the duplicated earth would not be counteracted

* The increase in intensity at the surface is as the cube root of the increase in the mass. (See page 45.)

by any increase in the mass of the moon; therefore the moon would immediately commence to fall towards the earth, and this motion of approach towards the centre of gravitation, being compounded with the moon's orbital motion, would cause an increase in the actual areal velocity; and the lessened distance would occasion a much greater apparent increase in the angular velocity; the result would be therefore that, when the centrifugal force had been intensified to an equality with the gravitating force, the moon would revolve with a considerably increased velocity at a less distance from the earth. As we have been brought to the conclusion that an addition to the mass of a planet revolving with a definite velocity at a definite distance from the centre of gravitation, will neither cause alteration in the velocity, nor in the orbital distance of the planet, and that this will be true even if the addition to the mass be very great; it will necessarily follow, by the same reasoning, that the converse of this will be also true, and if, instead of a large addition being made to the mass of the planet, we suppose a large deduction to be made from the mass, (supposing, for instance, the one-half of the planet to be removed,) the velocity of the planet's orbital motion and the distance of the planet from the centre of gravitation will remain unchanged, because the deduction from the planet's gravitating force is equally a deduction from its centrifugal force, and the same definite velocity determines the distance as before. And again, if we suppose an increase in the density and a proportionate decrease in the volume of the planet in consequence of contraction; or the reverse,—a decrease in density and increase of volume in consequence of expansion,—yet still the velocity and distance of the planet will remain unchanged; because in either of these cases the quantity of matter exerting gravitating influence previous to the contraction or to the expansion remains the same after the contraction or expansion has taken place, and therefore both the collective

gravitating force and the collective centrifugal force remain as before.

It is, however, proper to point out that the foregoing in respect to the addition to, or deduction from, a planetary mass, although true as to the absence of direct effect or influence upon the planet's orbital motion and radial distance, is not strictly true in reference to the indirect effect which such a change would occasion. To appreciate this indirect effect, it is necessary to take into consideration that combination of aggregated masses of matter, connected together through gravitation, acting and reacting upon each other and controlling each other's motions and relative positions, known as the Solar system. This system may be here briefly described as consisting of a number of planets of various sizes, revolving at various distances in the same (or nearly in the same) plane, round their common centre of gravitation, the sun. If we consider such a system attentively it will become evident that the addition of any considerable mass to any one of the planets would have the effect of drawing together the entire system: that is, of causing all the the planets to approach the sun*. For example take E, (Fig. 15) and suppose a considerable addition to the mass of that planet; the increase of gravitating force therefrom would act practically as an addition to the gravitating force of the sun. The first effect might be considered to be the attraction of those planets on the opposite side (or nearest to the opposite side) causing them to approach the sun; but the attraction of those planets would react on E itself, causing it also to approach; and the united effect of their attractions would evidently act on those at the other two sides, with a compounded influence just as though a single influence

*Note. It may be remarked that, unless such an addition were of a very great mass, the effect here spoken of would be rather a tendency than an appreciable effect; it would indeed be an actual effect, but so exceedingly small in amount as to be inappreciable.

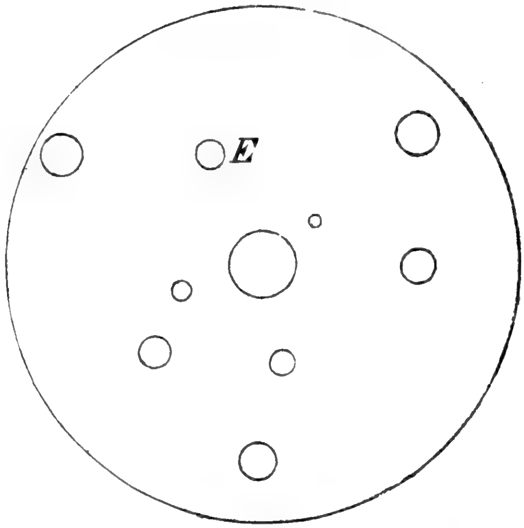


Fig. 15.

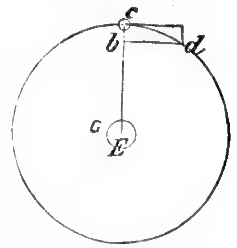
proceeded directly from the centre of the sun. So soon as any actual motion towards the sun took place, the increased velocity, and consequently increased centrifugal effect, would restore the equilibrium.

An addition to the mass of the sun itself, or an increase in the intensity of the gravitating influence, would have the like effect; by acting directly on the planetary bodies belonging to the solar system, causing them to approach the sun and to revolve with increased velocity. The entire system, in such case, would contract and occupy less space with a proportionate acceleration of motion amongst its members. In considering the general arrangement of the solar system, it will now become apparent that alterations of velocity in the orbital revolution of the various planetary members, are not dependent (or consequent) upon differences in their densities; and notwithstanding that some are vastly greater in magnitude than others, and situated at vastly greater or lesser distances from the centre of gravitation, there is not

necessarily a difference in their areal (actual) velocities; the difference in the distance being compensated by the greater or lesser angular (apparent) velocity, and the difference in mass or quantity of matter, producing a proportionate difference in the amount of centrifugal force and thereby counteracting and equalizing the greater or lesser amount of gravitating force. It is evident, however, that a planet of small size revolving at a very great distance from the sun, and with a consequently very small angular velocity, would be liable to excessive perturbation from the influence of other planetary bodies, and we should therefore be led to expect that the planets situated at the greatest distance from the sun would be the largest; and this is what actual observation shows to be the fact. We may now understand the unchangeable or permanent nature of the arrangements in respect to uniformity of the periodic revolutions, securing each planet from those changes in its velocity of motion and relative position which might otherwise have been occasioned by alterations in its own internal conditions. A planet having had, at the first, a definite velocity imparted to it, and a definite distance from the sun appointed for its orbital path, cannot deviate nor be readily made to depart therefrom. An outside influence, as that of another planet, will cause a vibration or oscillation in its motion, resulting in a more or less elliptical orbit; but any deviation in the one direction is necessarily followed by a compensating equivalent movement in the opposite; and the average distance from the sun remains the same. And, as already explained, changes in its own physical conditions, such as contraction or expansion of its volume, will not cause alteration in the velocity of its motion or of its position relatively to the other members of the system.

THE LAW OF GRAVITATION, AND THE NEWTONIAN THEORY.

We find the following in *Drew's Manual of Astronomy*, page 149: "Assuming, as the measurements of astronomers justify us in doing, that the moon is distant 240,000 miles from the earth we can easily calculate the space through which she would fall if left to herself in the time of a minute. This space would be as Newton has demonstrated in the 38th proposition of the Principia, the versed sine of the arc described in that time; thus suppose the arc Cd (Fig 29) to be that described in a minute by the moon C; join C to E the centre of the earth; draw bd perpendicular to CE; then will Cb, the versed sine of the arc, be the space through which the moon would fall if unsupported, that is if the centrifugal force should cease. Now the arc Cd is easily found, for as the moon takes 27 days, 7 hours, 43 minutes to describe her whole orbit, the following proportion will give it. As 27 days, 7 hours, 43 min: 1 min : : 360°: 33″ nearly=Cd: of this arc, the versed sine Cb. may be easily computed; it is in fact $16\frac{1}{12}$ feet, taking the moon's distance to be 240,000 miles. Now, on the supposition that the force retaining the moon in its orbit be identical with terrestrial gravity, it ought to have decreased in proportion to the square of the distance; that is to say, assuming the moon to be sixty times further from the earth's centre than a body on the earth's surface, or distant one semi diameter (that is, EC=60×EG)



which is about the truth: the space described by the moon should be to the space described by a falling body near the earth's surface as 60^2 to 1^2, the time being the same. Now a body would fall through 59,400 feet in one minute near the earth's surface; hence, as $60^2 : 1^2$ or as 3600 : 1 : : 59,400 feet : $16\frac{7}{9}$ feet* which is the space through which a body would fall in a minute at the distance of the moon. Now this exactly agrees, making allowance for our using round numbers, with the actual distance through which the moon would fall were the centrifugal force to cease. The moon, therefore, is retained in her orbit by gravity, and gravity only, for it would be unphilosophical to assign two causes to account for effects precisely similar."

This example, which at first sight appears to confirm the assumption that the attractive force or influence of gravitation is inversely as the square of the distance—an example indeed which is considered to stand almost in the place of an observed fact, and upon which the celebrated theory is in a great measure based,—does not, when closely examined support the assumption that the intensity of the influence is inversely as the square of the distance, but on the contrary, it proves that such assumption is quite irreconcileable with the facts; and it also goes far to demonstrate that, *in fact*, the gravitating influence (or attractive force) varies inversely as the distance simply; that is to say, that the gravitating influence decreases in equal proportion to the increase in the distance. The force of gravitation at the earth's surface causes a body falling freely to descend through $16\frac{1}{12}$ feet in a second. The moon is found to be by astronomical observation sixty semi-diameters of the earth distant from the earth (*i. e.* about 240,-000 miles.) The calculation then shows that at the dis-

* Note. There is evidently here an arithmetical error in the computation; it should read 57,900 : $16\frac{1}{12}$ feet.

tance of the moon a body would fall towards the earth through $16\frac{1}{12}$ feet in a minute. Now there are 60 seconds in a minute, and the distance of the moon from the earth is 60 times as great as the distance of the surface of the earth from the earth's centre. Therefore as the moon's distance : distance of the surface of earth : : attractive force at earth : attract force at moon. (or, inversely, as $1\ \frac{\text{diameter}}{2} : 60\ \frac{\text{diameters}}{2}$.)

The oversight which has vitiated the calculation quoted by Mr. Drew is the non-recognizance of the law of accelerated motion in its application to falling bodies. The absolute necessity of taking into consideration this law, because of its direct and important application in this case, cannot fail to be immediately observed when the attention is expressly called to it, although not obvious in the first instance. In the calculation and its result as given by Mr. Drew, we have: "A body would fall through 57,900 feet in one minute near the earth's surface; hence as $60^2 : 1^2$, or as 3600 : 1 : : 57,900 feet : $16\frac{1}{12}$ feet which is the space through which a body would fall in a minute at the distance of the moon." Let us test this result by inverting the case and assuming the force of attraction so obtained at the distance of the moon; let us see what intensity of force the application of the theory would in that case give us as the proportionate force at the surface of the earth. (Note. It may be here remarked that the law of accelerated motion (or velocity) in falling bodies has been very carefully investigated, and the results verified by numerous experiments of a reliable character. (The calculation just now stated, being inverted,—gives . . . .

$$\underbrace{1^2}_{1} : \underbrace{60^2}_{3600} :: \underbrace{\text{Intensity at Moon's distance}}_{16\frac{1}{12}\text{ feet in a minute.}} : \underbrace{\text{Intensity at Earth's surface.}}_{16\frac{1}{12}\text{ feet in }\frac{1\text{ second}}{60}}$$

Now $16\frac{1}{12}$ ft. in a minute, increased to 60 times the intensity, would (by the law of accelerated motion) be

$16\frac{1}{12}$ ft. in a second; but since we have 60 × 60, this must be again increased 60 times; and, consequently we have $16\frac{1}{12}$ feet in the 60th part of a second, as the measure of effective gravitating influence at the surface of the earth; or taking the result as space fallen through in a minute, we have 3564000 feet; that is to say, the actual intensity of gravitation at the earth's surface is thus enormously exaggerated and made to appear sixty times greater than it is *in fact.* The actual proportion (i.e. the true proportion) may be thus simply stated: As 1 $\frac{\text{diameter}}{2}$: 60 $\frac{\text{diameters}}{2}$ :: 1 second : 60 seconds; which (inversely) gives the gravitative intensity at the moon's distance. Or, As 1 second : 60 seconds :: 1 $\frac{\text{diameter}}{2}$: 60 $\frac{\text{diameters}}{2}$; which gives the distance of the moon from the centre of the earth.

THE RELATIVE MASS OF THE SUN TO THAT OF THE EARTH.

From Drew's Manual of Astronomy, page 174: "Let us first ascertain the deflection of the earth from a tangent caused by the sun's attraction. Proceeding in the same way as in that already pointed out for ascertaining the deflection of the moon produced by the earth's attraction (viz., C b in Fig. 29,) we shall find that the proportion between the distance through which the moon will be drawn by the earth, and that through which the earth will be drawn by the sun in the same time, will be as 1 : 2.2. Now, as we have already seen, the whole amount of attraction is in a ratio compounded of the ratios of the masses directly, and of the squares of the distance inversely; that is, letting F stand for the attractive force of the sun measured by the versed sine of the arc which the earth describes in one minute of time, viz. 2.2; and f, for the attractive force of the earth on the moon measured by the versed sine Cb, through which the moon would be drawn in a minute of time, viz., unity or 1; also the ratio of D to d, for that of the distances of the earth from the sun, and of the moon from the earth, which is as 400 : 1; and M to m, for the ratio of the masses of the sun and earth, which we desire to know; then $m : M :: f\,d^2 : F\,D^2$, that is, $m : M :: 1 \times 1^2 : 2.2 \times 400^2$, or, $m : M :: 1 : 352{,}000$. So that the mass of the earth is to that of the sun as 1 : 352,000; or it would take 352,000 earths to make a body equal in bulk to the sun."

It is evident that in this calculation an error requires correction of the same kind as that which so enormously exaggerated the result of the investigation as to the intensity of the gravitating influence at the distance of the moon. The corrected proportion will be

m : M :: fd : FD, and therefore, as 1 × 1 : 2.2 × 400, m : M :: 1 : 880. So that the mass of the earth is to that of the sun as 1 to 880. This is (or would be) a correction of the above computation, as it stands, assuming the elements of that computation to be correct; but upon endeavouring to verify those elements, we find ourselves unable to accept the quantity stated to represent the relative attractive force of the sun's influence on the earth compared to that of the earth's on the moon :—viz., 2.2 : 1.—because it is not apparent to us how or whence this quantity has been obtained; and besides this, the computation is based on a hypothesis . . that the distance to which a definite amount (or intensity) of gravitating force extends varies as the *mass* of the central or gravitating body . . . a hypothesis, which we hold to be quite untenable. (The distance varies as the cube root of the mass; See page 45). We therefore refer to another authority on the same subject.

Herschel's outlines of Astronomy, page 225.

(358) "That at so vast a distance the sun should appear to us of the size it does, and should so powerfully influence our condition by its heat and light, requires us to form a very grand conception of its actual magnitude, and of the scale on which those important processes are carried on within it, by which it is enabled to keep up its liberal and unceasing supply of these elements. As to its actual magnitude we can be at no loss, knowing its distance, and the angles under which its diameter appears to us. An object placed at the distance of 95,000,000 miles, and subtending an angle of 32′ 1″, must have a real diameter of 882,000 miles. Such, then, is the diameter of this stupendous globe. If we compare it with what we have already ascertained of the dimensions of our own, we shall find that in linear magnitude it exceeds the earth in the proportion 111½ to 1, and in bulk in that of 1,384,472 to 1."

(360) "Admitting, then, in conformity with the laws

of dynamics, that two bodies connected with and revolving about each other in free space do, in fact, revolve about their common centre of gravity, which remains immoveable by their mutual action, it becomes a matter of further enquiry, *whereabouts* between them this centre is situated. Mechanics teach us that its place will divide their mutual distance in the inverse ratio of their *weights* or *masses*; and calculations grounded on phenomena, of which an account will be given further on, inform us that this ratio, in the case of the sun and earth, is actually that of 354,936 to 1. . . . the sun being in that proportion more ponderous than the earth. From this it will follow that the common point about which they both circulate is only 267 miles from the sun's centre,* or about $\frac{1}{3300}$th part of its own diameter.

We observe that the quantity, given in the last section as the weight of the sun compared to the earth, corresponds very closely to that given by Mr. Drew, as the proportion of the mass or bulk. The computation (correct data being obtained) might be made in the manner which Mr. Drew seems to have intended; observing that the difference in the distance is compounded with a difference in the areal as well as the angular velocity. Taking the distance of sun and earth, and earth and moon, as 400: 1. Taking the moon's revolution at 28 days, and the earth's at 365 days, as (about) 13 : 1. We have ($\frac{400}{13} = 30\frac{10}{13}$) 31 : 1, as the areal velocity of the

* This teaching—viz., that the earth and sun revolve around a common centre of gravity, at a certain distance from the sun's centre, may be considered true in a theoretical sense, by discarding the compensating influence of all the other planetary members of the system. To obtain rigid demonstration would require a very perfect knowledge of the solar system, but, even with the knowledge we now possess, there can scarcely be a reasonable doubt that the centre of the sun is the actual centre of revolution of the system; and, relatively to the members, separately and collectively, belonging to that system, the centre of the sun therefore may be correctly considered as a fixed or immovable point.

earth (compared to the moon's), and 1 : 13 as the angular velocity. But the effective (gravitating,) influence is inversely as the distance divided by the difference † in angular velocity ; therefore $\frac{400}{13} = 31$. And the computation becomes $31 \times \frac{31}{13} =$ (about) 72 which quantity should represent the sun's diameter compared to the diameter of the earth as unity, assuming ihe *densities* of the sun and earth to be equal. Herschel's result from the comparison of densities and the direct measurement of the sun's diameter is, as above, 354936 : 1, as the proportion of the volumes (masses) at an equality of density. The cube root of this quantity . . . $\sqrt[3]{354936}$ . . . equals (very nearly) 71.—So that the two results agree quite as closely as can be expected, considering the inexactitude of the measurements taken. It may be remarked that, in the foregoing comparison of the relative masses and densities of the earth and the sun, the atmosphere of the earth is entirely left out of consideration; but evidently the atmosphere is a part of the earth, considered (collectively) as a planet,. . . .a part of the mass and of the volume. . . .a part of the density. . . .and (contributing) a part of the gravitating influence. Therefore the above comparison should be defined as, between the solid and liquid parts only of the earth . . and the entire volume of the sun.

The Third of Kepler's laws.

From Drew's Manual of Astronomy, page 177 : "The squares of the periodic times of any of the planets are to each other as the cubes of their mean distances from the sun.

The value of this law will appear when we consider that knowing the distance of any one planet, say for instance the earth from the sun, we can by its aid calculate the distance of any other." The truth of this

† Divided by the proportionate angular velocity if less than at the lesser distance, and inversely as the distance multiplied by the proportionate a. v. if greater than at the lesser distance.

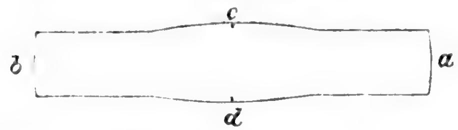
law must be in the first place dependent upon the areal velocity of all the planets being the same, because evidently a difference in actual velocity, would cause a difference in the periodic times, which would have no direct or necessary relation to the planet's distance. From what has been already explained as to the necessity of an equality between the gravitating and centrifugal forces, it follows that there is a velocity of equilibrium for each definite distance from the sun; and which velocity of equilibrium cannot be departed from. And this velocity of equilibrium must be the same actual areal velocity whatever may be the distance of the particular planet from the centre of gravitation; because both the centrifugal and gravitating forces are alike dependent upon and proportionate to the actual velocity. Therefore Kepler's third law as to such relationship is based upon fact. [The mistake as to the actual relation may have very probably originated in neglect to discriminate between the angular and the areal velocities: presenting itself to him as an induction from actual observation, and occupying the place of a demonstrated fact, it doubtless had much effect in misleading Newton in his theory of gravitation.] Since it now appears that the variation of the gravitating influence is inversely equal to the variation in the distance, it follows that the relation of the periodic times and the distances is also simple; and accordingly the law (*i.e.* the third law of Kepler) should be read thus:—'The periodic times of any of the planets are to each other as their mean distances from the sun.'

THE FORM OF THE EARTH AND TERRESTRIAL GRAVITATION.

From Drew's Manual of Astronomy, page 25.

"This centrifugal force has caused matter to accumulate in the regions of the equator, so that the earth's equatorial diameter is greater than its polar by 26.5 miles. And since the attraction of gravity decreases in the inverse proportion of the squares of the distance from the centre of the attracting sphere, it is plain that the attraction will be weaker on the earth's surface near the equator than at or near the poles in the proportion of the square of half the earth's equatorial diameter to the square of half the polar diameter." This statement would have much force if the earth's gravitation was actually located at and acted from a point or small spherical place situated at the earth's centre. For many purposes it is convenient and does not practically involve any error to refer the aggregate or collective action of gravitation to the centre; since if we assume the earth to be a perfect sphere the attractive force would be everywhere on or beyond the surface equivalent to the effect of an influence apparently situated at the centre, but it is necessary to recollect that the central force so assumed is representative (because compounded) of the sum of the forces of all the parts or fractions of which the earth is composed: for evidently if the exterior parts of the earth were removed and only a small central sphere left remaining, the attractive force would be diminished accordingly. The error may be immediately and distinctly seen by exaggerating the deviation from a sphere and supposing the earth to be flattened into a disc-like form (*i.e.* the form of a grindstone) as in Fig. 16, when

a, b would represent the equatorial diameter and c, d the polar diameter. Now assuming the earth to be of such form, and leaving the rotation on the axis out of the question, it is at once evident that the attractive force on the surface at a, and b, would not be less than at c, and d, but on the contrary would be considerably greater, which is the reverse of the conclusion stated in the text.



From Drew's Manual of Astronomy, page 23 : "It is found, however, by very careful admeasurement, that the length of a degree of latitude, ascertained in the manner described, near the equator, is different from the length of a degree measured near the poles; on the equator it is at its minimum, increasing in length as the latitude increases. In fact a section of the earth passing through both poles would not be a circle, but, as the admeasurements show, would be an ellipse; indicating that the earth is flattened at the poles, and that it protrudes in the region of the equator. The first intimation which astronomers received of this fact arose from the following circumstances. Astronomers sent in the seventeenth century by the French government to Cayenne, for the purpose of making observations on the fixed stars, found that their clocks, the pendulums of which had been so regulated as to beat seconds in the latitude of Paris, lost time at the rate of two minutes twenty-eight seconds per day. When this fact became generally known, it deeply interested the leading mathematicians in Europe, who applied themselves diligently to account for this variation; Huygens and Newton simultaneously discovered the cause."

But the reasoning applied to the observed facts is not correctly based on the recognized laws of physical science, and the conclusions, being in a scientific sense unsupported, are therefore unreliable.

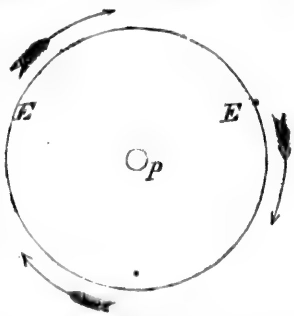
From Drew's Manual of Astronomy, page 26: "Sir I. Newton, taking both these causes into account, demonstrated that a revolving fluid mass of equal density throughout, would assume the form of an ellipsoid [that is a figure of which every section passing through the poles would be an ellipse] whose diameters would be as 230 : 229."

[The rapidity of revolution (rotation) is not here stated or referred to; a greater or lesser rapidity would certainly cause (a theoretical) alteration in the proportions of the relative diameters. No doubt the case supposes the present rate of rotation of the earth]. Admiting the demonstration; does it not follow that, if the earth had precisely the ellipsoidal form, there would be no active or positive centrifugal effect? Because, to suppose or assume a centrifugal effect on the surface of the solid, or partially solid, earth; evidently implies that an alteration in the form would take place if the fluid condition was substituted for the solid; or in other words, if the particles of matter had the necessary mobility. It is stated, however, that the actual difference between the two diameters is less than Newton's calculation required; the proportions derived from measurements of sections of the earth's circumference —*i. e.* degrees in various latitudes from 0° to 80° inclusive,—being as 298 : 299, instead of 229 : 230 as required by the calculation. This would give a (an effective) centrifugal force limited for its originating influence to (299.21--299) $\frac{21}{29900}$ of the mean diameter of the earth, a quantity which may be sufficient to account for the observed facts in respect to the pendulum vibrations.

NOTE.—*But it must be observed that an assumed correct measurement of the earth's diameter derived from measurement of the earth's surface, assumes the correctness of the supposed "ratio of the diameter to the circumference of the circle," a question at the present time in controversy.*

THE ROTATION OF THE EARTH (OR OTHER PLANET) AS INFLUENCING TERRESTRIAL (INTERNAL) GRAVITATION.

An apparent difficulty may be thus stated. Let Fig. 18 represent half of the earth's sphere; E, E, E being a section at the equator, and P the place of one of the poles. The earth is rotating in the direction of the arrows, and a body on the surface at E, is therefore revolving with that surface, around the centre of the earth at the rate of about 1000 miles an hour. A body on the surface at or near P, is comparatively uninfluenced by this rotatory motion. An argument to the following effect is therefore likely to suggest itself to the student: (1) A body revolving around a centre of gravitation (or central point) at the rate of a thousand miles an hour must develope a very considerable centrifugal force. (2) This centrifugal force opposes the attractive force from the centre, and must therefore (3) be a deduction from the apparent gravity or weight of the body. Hence (query), is the (apparent) weight of the same body considerably greater in the polar regions than near the equator? or is there any sensible difference in the (apparent) weight of the same body if removed from the one to the other situation? The question as to fact is answered in the negative:—where is the explanation?

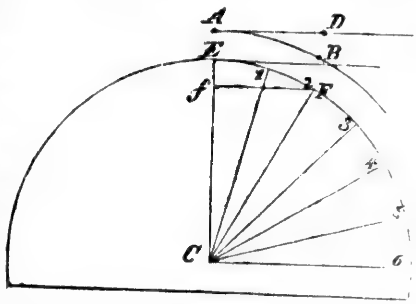


The great stumbling block in this case, is the same already referred to as the probable origin of the error in Kepler's third law; viz., the neglect to discriminate

between angular and areal velocity. It is true the body on the surface near the equator has an actual areal velocity in the circle of rotation of about 1000 miles an hour; nevertheless the motion is correctly represented by the hour-hand of a watch, which likewise passes through the circle once in 24 hours; and if the hour-hand were extended and made about 4000 miles in length, the extremity of the hour-hand would also then move with a velocity of about 1000 miles an hour. So soon as this fact is correctly appreciated the difficulty will probably be in a great measure removed, because it will be understood that the angular velocity as observed in the hour-hand of the watch or clock is not sufficient to develope any very appreciable amount of centrifugal force. The difficulty may, however, present itself in a different form, and one in which the solution is not so readily apparent. A body revolving at a short distance from the surface around the earth with such velocity as to make a complete revolution in a little less than 11 hours would not fall to the earth but continue to revolve in its orbit at that distance in the same manner as the moon. (Note. This is of course discarding the earth's atmosphere;—*i. e.* supposing the earth to be without an atmosphere; which would resist and impede the motion of the body.) But then, if the revolution performed in 11 hours is sufficient to neutralize and overcome entirely the attractive force; must not a revolution of 24 hours, which represents nearly one half the velocity, have at least some considerable influence in reducing the effect of the central gravitating force? To explain this case, it is proper in the first place to point out that the term, centrifugal force, is strictly speaking a misnomer. It is one of those names which contains within itself a definition, and that definition is mischievous because it is false. The force (and the motion out of which the force is developed) is not centrifugal, but tangential. The teaching on this subject authorized at the present time would, directly, or indirectly, lead the student to the conclusion that the attractive force, acting on a body in motion at right angles to the direction

in which the body is moving, combines with, interferes with, increases or retards the motion of the body, and so becomes partially or wholly inoperative in the direction of the centre whence it proceeds. It has been shown in the earlier part of this lecture that such teaching is unsound and therefore untenable. In fact the gravitating (or attractive force) is not interfered with nor diminished by the tangential motion of the revolving body; for example, the moon is constantly acted upon with the full force of terrestrial gravitation, and the effect of that force is to cause the moon to constantly deviate from the tangential direction of motion. Consequently if a body revolves round the earth near the surface at the equator, it is subject to the full attractive force of gravitation; and if it falls to the earth, so soon as it reaches the ground its independent motion must cease, and its gravity or weight is the same as at any other place on the surface. The question as to whether a body revolving near the surface can escape contact and continue to revolve, is evidently dependent upon equality between the versed sine of the arc through which the body moves in a definite time, and the distance through which the gravitation would move the body towards the centre in the same definite time; because the latter measures the former and, if equal, causes no more than the deviation from the tangential direction, which is required to produce the arc, and in that case the body will continue to revolve; but if the deviation is in any degree greater than requisite for the versed sine of the arc, then must the body fall to the earth. By taking the equivalent to a thousand miles at the equator on a circle, the case may be illustrated,—as in fig. 19, where the angular divisions 1 2 3 4 5 6 are each equivalent to about 1000 miles; and the body A, near the surface of the earth is supposed to be moving in the tangential direction AD, with a velocity of about 36½ miles in 1 minute; (which would carry it through a space equal to the circumference of the earth in about 10 hours and 57 minutes.) The effect of terrestrial gravitation acting freely on a body

near the surface is known to equal in effect an approach towards the centre (or space fallen through) of 57,900 feet in a minute. This would be therefore the deviation from the tangential line of motion caused in the body A by the attractive force; viz, (about) 1586 feet in 5280 feet. (equal to about 600 miles in 2000 miles.) The body A, would consequently travel 2000 miles in 54¾ minutes, and arrive at B, having in that same time undergone a deviation of its motion from the tangential line



AD, measured by the versed sine AE, of about 600 miles. And since the distance BF, is equal to the distance AE, the body A, will revolve continually in an orbit at that distance. (The velocity, however, should be a little increased to equalize the difference in the radial distance of CA greater than CE; (e g, if AE=10 miles; then CA: CE : : 4010 : 4000 miles.)*

THE TIDES.

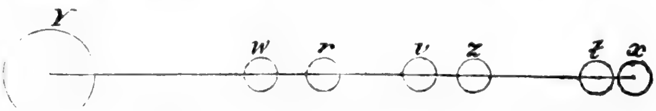
Drew's Manual of Astronomy. Page 78.

"*In fig* 45 *plate VI*, let Z represent the moon, R the

earth. Now the moon attracts every particle of the earth; and the water being free to move, will tend towards her at o : it will be high tide, therefore, to those places situated at o and its neighbourhood, which have

* The supposition of such increase is included in the assumption above that the distance BF, is equal to the distance AE.

the moon on the meridian; but since the quantity of water remains the same, the places at n and s, 90° distant from o will supply the rise at p; with them therefore and down the line n R s it will be low water. As the earth turns round with her diurnal motion, other places will advance towards the moon, or will have her in the meridian; it will therefore be high tide to them at that time. So far the matter is clear; but the peculiarity is, that when it is high tide at o it will be also high at q, diametrically opposite, or with those places on whose inferior meridian the moon is situated. To render our explanation of this fact more lucid, let us investigate the operation of attraction on three bodies, at different distances from the attractive body (Fig 12). The effect of a



body Y, operating on three others r, z, x, in the same line would be to increase their mutual distances; for r would be drawn to w, through the space rw; z, being further off from Y, would be drawn through a less space, in the proportion $Yr^2 : Yz^2$, viz., to v, zv being less than rw; x would be still less operated upon, and would pass through a less space towards the attracting body; viz. xt. The result will be, that the distance of the two bodies r and x, from z, will be increased; vw and vt, their new distances being greater than zr and zx, their original distances. Let the waters on either side of the earth R, in fig. 45, pl. vi, be considered in the same circumstances as the two bodies x and r with respect to z in fig 12. The operation of the attraction of the moon z, upon them and the earth will be to raise the waters at p, and to draw the earth, as it were away from the waters at r, causing a simultaneous rising of the tides at o and q." The explanation here given is that the moon attracts the water on the earth's surface, at

the side next to her, and draws it from the earth towards her, and in addition attracts the earth itself, and draws that also from the water on the opposite side; thus accounting for the high tide at both sides; but if such were the case the earth and moon would soon come together; because, if the larger body,—the earth, deviates from her orbit to approach the moon, the lesser body,—the moon, must deviate so much the more (in inverse proportion to the mass) to approach the earth; and,as this approximation is supposed to be continuous, the result would necessarily be, within a certain limited time, contact between the earth and the moon. The hypothesis, that the water is drawn towards the moon at the side opposite to her, does not when attentively considered appear to be tenable; no reason is shown why such an effect should be confined to the water only; the air would be proportionately subject to the same action; and solids on the earth's surface also; variations in the barometric level and in pendulum vibrations would indicate and measure such a partial and local effect of the moon's gravitating influence. Such a cause operating in the slightest imaginable degree, in the manner supposed, would have the effect of accumulating all the water on the earth's surface, in that part exposed to the moon's direct influence. Moreover the sun's attraction is supposed to operate in the same manner. Page 79 *(Drew's Astronomy).* "Not only is the moon an agent in producing tides, but the sun also; in consequence, however, of his greater distance his attraction is not so much felt; the whole force of attraction being in compound proportion of the mass directly and the distance squared inversely. The force of attraction thus deduced will give the sun's attraction : the moon's : : 2 : 5."

The hypothesis is negatived by the fact that the influence of the solar gravitation on the earth's surface is the same by night as by day. Substances, whether solid or fluid, weigh the same at midnight as at midday whereas if the hypothesis were sound there would necessarily be a difference, and the apparent gravity of each thing on the

earth's surface would be greater at night, and less by day.

OF THE TIDAL PHENOMENA

a reasonable explanation may be found in the form of the earth as deviating from an ellipsoid and in (conjunction therewith) the unevenness of the earth's surface. As already stated, assuming Sir I. Newton's calculation of the perfect planetary ellipsoid, and the correctness of the actual measurements of the polar and equatorial diameters, the difference of $\frac{21}{22900}$ indicates a cause of inequality; which would allow the rotatory motion to occasion a positive centrifugal effect in the equatorial regions; and granting any effect, however slight, from this cause, the quantity of effect would be in a measure dependent upon the conformation of the earth's surface (and upon local differences in the conformation) greater or less in different places according to the depression or elevation of the surface at those places. And moreover, the manner in which the effect would manifest itself would be to some extent determined or modified by the particular configuration of the uneven surface, (i. e. the part of the surface covered with water; viz., the elevations and depressions in the bed of the ocean). It is not difficult to understand how an oscillatory effect upon the waters of the ocean thus produced,—being modified, increased or lessened, by the action of such extra causes as winds, aerial and ocean currents, the unequal influence of light, heat, &c.,—may result in what is known as the ebb and flow of the tide (and of the other tidal phenomena). The explanation of the double tide is to be found in the law regulating the equilibrium of bodies rotating on a central axis; when a part of the matter composing the body is free to move, and so prevent a disturbance of the centre of gravity by compensating for an alteration in the quantity of matter (or the quantity of gravitating effect) on the one side, by increasing or diminishing the mass equally on the opposite side.

ANGULAR AND AREAL VELOCITIES.

Apparent motion of the sun. *From Herschel's Outlines of Astronomy*, pages 219 to 222

§ (347). "We have already seen (art. 146) that the sun's motion in right ascension among the stars is not uniform. This is partly accounted for by the obliquity of the ecliptic, in consequence of which equal variations in longitude do not correspond to equal changes in right ascension. But if we observe the place of the sun daily throughout the year, by the transit and circle, and from these calculate the longitude for each day, it will still be found that, even in its own proper path, its apparent angular motion is far from uniform. The change of longitude in twenty-four mean solar hours *averages* $0^\circ\ 59'\ 8''.33$; but about the 31st December it amounts to $1^\circ\ 1'\ 9''.9$, and about the 1st of July is only $0^\circ\ 57\ 11''.5$. Such are the extreme limits, and such the mean value of the sun's apparent angular velocity in its annual orbit.

§ (348). This variation of its angular velocity is accompanied with a corresponding change of its distance from us. The change of distance is recognized by a variation observed to take place in its apparent diameter, when measured at different seasons of the year, with an instrument adapted for that purpose, called the heliometer, or, by calculating from the time which its disc takes to traverse the meridian in the transit instrument. The greatest apparent diameter corresponds to the 1st of January, or to the greatest angular velocity, and measures $32'\ 36''.2$; the least is $31'\ 32''.0$, and corresponds to the 1st of July; at which epochs, as we have seen, the angular motion is also at its extreme limit either way. Now, as we cannot suppose the sun to alter its real size period-

ically, the observed change of its apparent size can only arise from an actual change of distance. And the sines or tangents of such small arcs being proportional to the arcs themselves*, its distances from us, at the above-named epoch, must be in the inverse proportion of the apparent diameters. It appears, therefore, that the greatest, the mean, and the least distances of the sun from us are in the respective proportions of the numbers 1.01679, 1.00000, and 0.98321; and that its apparent angular velocity diminishes as the distance increases, and *vice versâ*."

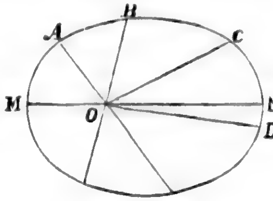
§ (349). "It follows from this, that the real orbit of the sun, as referred to the earth supposed at rest, is not a circle with the earth in the centre. The situation of the earth within it is *excentric*, the *excentricity* amounting to 0.01679 of the mean distance, which may be regarded as our unit of measure in this enquiry. But besides this, the form of the orbit is not circular, but elliptic."

§ (350). "The mean distance of the earth and sun being taken for unity, the extremes are 1.01679 and 0.98321. But if we compare, in like manner, the mean or average angular velocity with the extremes, greatest and least, we shall find these to be in the proportions of 1.03386, 1.00000, and 0.96670. The variation of the sun's *angular velocity*, then, is much greater in proportion than that of its distance, fully twice as great; and if we examine its numerical expressions at different periods, comparing them with the mean value, and also with the corresponding distances, it will be found, that, by whatever fraction of its mean value the distance exceeds the mean, the angular velocity will fall short of its mean or average quantity by very nearly twice as great a fraction of the

* The meaning of this paragraph is not very apparent. The sines or tangents of all arcs are proportional to the arcs, whether large or small. Has not Sir John intended to say that the sines or tangents of very small arcs nearly correspond with (i.e., are nearly equal to) the arcs themselves?

latter, and *vice versâ*. Hence we are led to conclude that the *angular velocity* is in the inverse proportion, not of the distance simply, but of its square; so that, to compare the daily motion in longitude of the sun, at one point A, of its path, with that at B, we must state the proportion thus:—

$OB^2 : OA^2 : :$ daily motion at A : daily motion at B. And this is found to be exactly verified in every part of the orbit."



(Art. 349.)

§ (351). "Hence we deduce another remarkable conclusion, viz.,—that if the sun be supposed really to move around the circumference of this ellipse, its actual speed cannot be uniform, but must be greatest at its least distance and less at its greatest. For, were it uniform, the apparent angular velocity would be, of course, inversely proportional to the distance; simply because the same linear change of place, being produced in the same time at different distances from the eye, must, by the laws of perspective, correspond to apparent angular displacements inversely as those distances. Since, then, observation indicates a more rapid law of variation in the angular velocities, it is evident that mere change of distance, unaccompanied with a change of actual speed, is insufficient to account for it; and that the increased proximity of the sun to the earth must be accompanied with an actual increase of its real velocity of motion along its path."

§ (352). "This elliptic form of the sun's path, the excentric position of the earth within it, and the unequal speed with which it is actually traversed by the sun itself, all tend to render the calculation of its longitude from theory, (*i. e.*, from a knowledge of the causes and nature of its motion) difficult; and indeed impossible, so long as the law of its actual velocity continues unknown.

This law, however, is not immediately apparent. It does not come forward, as it were, and present itself at once, like the elliptic form of the orbit, by a direct comparison of angles and distances, but requires an attentive consideration of the whole series of observations registered during an entire period. It was not, therefore, without much painful and laborious calculation, that it was discovered by Kepler (who was also the first to ascertain the elliptic form of the orbit) and announced in the following terms:—Let a line be always supposed to connect the sun, supposed in motion, with the earth, supposed at rest; then, as the sun moves along its ellipse, this line (which is called in astronomy the *radius-vector*) will *describe* or *sweep over* that portion of the whole *area* or *surface* of the ellipse which is included between its consecutive positions: and the motion of the sun will be such that *equal areas* are thus *swept over* by the revolving radius vector *in equal times*, in whatever part of the circumference of the ellipse the sun may be moving."

§ (353). "From this it necessarily follows, that in unequal times, the areas described must be proportional to the times. Thus, in the figure of art. 349, the time in which the sun moves from A to B, is to the time in which it moves from C to D, as the area of the elliptic sector AOB is to the area of the sector DOC."

§ (354). "The circumstances of the sun's apparent annual motion may, therefore, be summed up as follows: —It is performed in an orbit lying in one plane passing through the earth's centre, called the plane of the ecliptic, and whose projection on the heavens is the great circle so-called. In this plane its motion is from west to east, or to a spectator looking down on the plane of the ecliptic from the northern side, in a direction the reverse of that of the hands of a watch laid face uppermost. In this plane, however, the actual path is not circular, but elliptical; having the earth, not in its centre, but in one focus. The eccentricity of this ellipse

is 0.01679, in parts of a unit equal to the *mean distance*, or *half the longer diameter of the ellipse* ; *i.e.*, about one-sixtieth part of that semi-diameter; and the motion of the sun in its circumference is so regulated, that equal areas of the ellipse are passed over by the radius vector in equal times."

The conclusions here arrived at—viz., that the angular velocity varies inversely as the square of the distance, and not in simple inverse proportion—appears to be (is obviously) unreasonable; for example, let us suppose the distance of the earth from the sun to be suddenly reduced to the one-half, and that the earth continues still to travel in the orbit of revolution with the same velocity as before. Since the earth to make a complete circle of revolution will now have half the distance to travel through, and travels with the same speed as before, it is reasonable to infer that the time occupied would be the one-half (*i. e.* 182½ days). The above conclusion, or theory, would make it necessary for the earth to accomplish the half distance in one-fourth of the time, and this without any increase in the areal velocity of motion: (*i. e.* in the actual speed). The precise nature of the error, as herein manifested, it is not difficult to point out.—Sir John Herschel has omitted to observe that in reducing the distance, the standard of comparison is likewise reduced. An arc subtending any definite angle is still at the lesser distance a similar arc, but evidently the actual length is reduced, and one degree at the lesser distance is therefore, although it is still a degree, not a degree of the same length as the degree at the greater distance. Referring again to the example of the earth at the half distance from the sun, the lesser circle will contain 360 degrees, but since two of these circles would be only equal to the one circle at the greater distance, if the increased velocity be considered in comparison with the greater circle as the standard of comparison, the increase will be 50 per cent. ; but if compared with

the lesser circle as the standard, then it will be 100 per cent., and what is true in respect to the circle also applies to every (equal angular) division of the circle; as for example, to one degree.

In the succeeding section (§ 351) Herschel, apparently, attributes the increased velocity, to motion acquired in the approach of the earth to the sun.* In our previous calculation, page 35, the increase (in areal velocity) arising in this manner, was shown to be $\frac{1}{2260}$ (or about 1 mile in 2260 miles.) The theory of an increase of velocity inversely as the square of the distance would require an increase (in areal velocity) of $\frac{1}{30}$ (or 1 mile in thirty miles.

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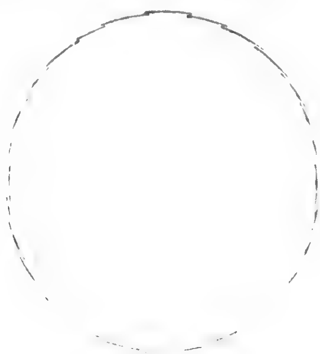
* Taking the concluding, together with the first sentence of this section (§351), the idea is suggested that Herschel was impressed by this supposed circumstance (or phenomenon) as being a remarkable and mysterious necessity, rather than as an effect satisfactorily accounted for or attributable to a recognised law.

THE ELLIPTICAL ORBIT, AND LAW OF EQUAL AREAS.

To test the soundness of Sir I. Newton's demonstration (in the Principia,) let us suppose a planet revolving in an orbit ot very great eccentricity, in which the major axis of the ellipse : minor axis : : two : one. At (about) aphelion therefore, the planet moves in a circle of which the diameter is to the diameter of the circle in which the planet moves when at (about) perihelion, as 2 : 1. It is evident that—supposing the planet, in revolving throughout a greater orbital circle at the distance of aphelion, completed the revolution in a certain definite time,—if made to revolve throughout a lesser circle at the distance of perihelion, it would complete a revolution in one half the time; or would make two revolutions in the same time in which it had made one revolution at the greater distance; but what is true of the entire circles is also true when applied to their fractional parts; therefore, if the planet moves through nine degrees of the greater circle (at aphelion) in any definite time (t), it will move in the same time (t) through eighteen degrees of the lesser circle (at perihelion). Now if the areas of circles varied as their diameters or as their perimeters; then would the areas passed over by the radius-vector in each case be eqnal; but *in fact* the areas vary as the squares of the diameters, and consequently the area swept by the radius-vector of the lesser circle is only the one half of the area swept in the same definite time (t) by the radius vector of the greater circle; because, evidently, for the areas to be equal, the planet would require to complete four revolutions of the lesser circle in the same time as one revolution of the greater circle, but that would be to move through twice the

space in the same time without increase of velocity, which is absurd; wherefore,—the supposed demonstration must be unsound.

An ellipse, it is true, differs from a circle, but the difference is not of such a kind as to affect the application of the same rule; in fact an ellipse may be most correctly considered as a circle described with a varying (*i.e.* an increasing and diminishing) radius; and therefore it comes under the same rule or law. Perhaps the most simple and satisfactory way to appreciate the difference between the circle and ellipse, and to exhibit the compound nature of the latter, is to form the ellipse from the fractions of a number of circles, of which the radii decrease by graduation to a certain limit (the minor axis), and then again increase to the former limit (the major axis) as shewn at Fig. 20, in which the ellipse is entirely composed of the fractional parts of circles.



HABITABILITY OF THE MOON.

From Herschel's Outlines of Astronomy, page 287.

"On the subject of the moon's habitability, the complete absence of air noticed in art (431), *if general over her whole surface* would of course be decisive. Some considerations of a contrary nature, however, suggest themselves in consequence of a remark lately made by Prof. Hansen, viz., that the fact of the moon turning always the same face towards the earth is in all probability the result of an elongation of its figure in the direction of a line joining the centres of both the bodies acting conjointly *with a non-coincidence of its centre of gravity with its centre of symmetry.* To the middle of the length of a stick, loaded with a heavy weight at one end and a light one at the other, attach a string, and swing it round. The heavy weight will assume and maintain a position in the circulation of the joint mass farther from the hand than the lighter. This is not improbably what takes place in the moon. Anticipating to a certain extent what he will find more fully detailed in the next chapter, the reader may consider the moon as retained in her orbit about the earth by some coercing power analogous to that which the hand exerts on the compound mass above described through the string. Suppose, then, its globe made up of materials not homogeneous, and so dispersed in its interior that some considerable preponderance of weight should exist excentrically situated: then it will be easily apprehended that the portion of its surface nearer to that heavier portion of its solid content, under all the circumstances of a rotation so adjusted, will permanently occupy the situation most remote from the earth."

The experiment (or example) upon which the foregoing hypothesis is based evidently does not apply to the con-

ditions and circumstances of the case under consideration; viz., the revolution of the moon round the earth, the relative positions of the two bodies and the nature of the force connecting them together. To enable the revolution of the stick loaded with weights to serve as an example or illustration it would, at the least, be proper for a great number of strings to be attached to all parts in the length of the stick from end to end; and then, if those strings were all possessed of a considerable and precisely equal amount of elasticity, the result of an experiment tried therewith would have a reasonable application to the case of the moon and the earth connected together by the influence of gravitation; but even *this* would not correctly represent or exemplify the conditions of the case here investigated; to obtain a demonstrative comparison with those conditions it would be necessary for the string to be attached to the centre of gravity of the combination (compound body), and *then* to show that, on being swung round, the end most heavily loaded assumes and maintains a position further from the hand than the opposite end. The result of such an experiment, properly conducted, is not, however, by any means doubtful; it would certainly be opposed to the hypothesis of Prof. Hansen; because the body, whether it were the arrangement compounded of the stick and weights, or the moon, would retain the position in which it might happen to be, or in which it was placed at the time the motion of revolution was first imparted to it; and, if an impulse of rotation was also communicated to it, it would continue to rotate around its own centre of gravity, whilst travelling in the orbital path of revolution around its primary centre of gravitating (or restraining) influence.

In the remarks as to the physical conditions on the moon's surface, Sir John Herschel supposes the existence of both water (in the liquid form) and of an atmosphere; all the water, and almost all the atmosphere, being confined

to the one hemisphere, in consequence of a superior gravitating force on that side furthest from the earth; but the form of the (solid) moon is not supposed to deviate very considerably from a sphere, and it is therefore not apparent how the hypothesis of the existence of the water and air on the one side only is to be reconciled with the circumstances of the case; for instance the average intensity of gravity on the surface of the moon compared to that of the earth is about 4 : 15 that is, if the earth's gravity is represented by 15lbs. on the square inch, that of the moon would be about 4lbs. on the square inch. Now the moon's atmosphere is supposed to be proportional in quantity to that of the earth; it would be, therefore, about one fourth of the depth (or heighth). Since 15lbs. on the square inch is the weight of the greater column, acted on by the earth's more intense influence; what will be the pressure caused by the lesser column only one-fourth the heighth, acted on by the (moon's) influence less intense in the proportion of 4 : 15.? When it is understood that, on the supposition of a uniform gravitating influence and a uniform distribution of water, together with the quantity of atmosphere conjectured, the water would be subjected to a pressure only the *one-fifteenth* of that on the surface of the earth; and that the air at the surface of the moon would be *far less dense* than at the summit of our highest mountains; the very large amount of concentration of gravitating influence which would be required in the one hemisphere to meet the requisitions of the hypothesis becomes apparent. We may suggest that, by supposing a much greater alteration of form, and assuming that the side of the moon furthest from us may be more or less concave instead of convex, the conditions would be then such that the existence of water in the liquid form and of animal and vegetable life (such as known to us) would be possible and not perhaps (violently) improbable. The effect would be to give such a concentration as the circumstances require. Fig. 21 (a) and

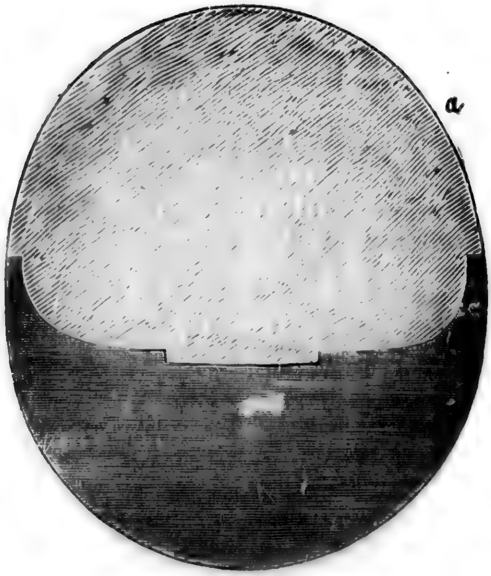


Fig. 21.

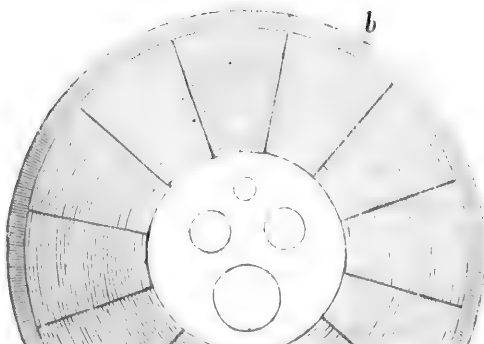


Fig. 21. b

(b) may serve to convey a generel idea of the arrangement supposed; (a) being a vertical section of the moon through the centre, (*i.e.*, by a plane passing through the centres of the moon and earth) and (b) a surface view or plan of the side furthest from us. The rainfall is supposed to take place on the high land forming the side of the concavity [which may be supposed to consist of a circular chain of rocky mountains] whence collecting into rivers and streams, and watering the intervening habitable country, it would flow into the central ocean, therein again to undergo evaporation from the influence of the sun and of the internal temperature. A great aerial current ascending in the central, and descending in the higher regions, near the sides [*i.e.* the circular boundary] would assist and regulate this effect, and the atmosphere itself would be concentrated and retained [at least in a great measure] within and above the concave side. The central ocean may be supposed to contain large islands as shown in the figure; to be entirely open; or, to be divided by land into several large lakes or seas. The diameter or breadth of the central ocean may be taken at about 800 miles; and the habitable land surrounding it to have a breadth of about 500 miles on each side; which would give in round numbers 500,000 square miles of ocean surface, and 2,000,000 square miles habitable land surface.

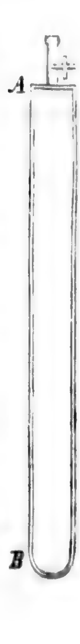
'GRAVITATION' AND THE 'ATOMIC THEORY.'

The expressions 'densities,' 'specific gravities,' 'atomic weights' may be used without a clear perception of the distinctions and relations between them. If a body in the solid condition, capable of undergoing compression, be subjected to a sufficient pressure, its bulk or volume will be diminished and a proportionate increase take place in its density; its weight or gravity, therefore, *relatively* to its *bulk*, will have become greater. Hence, specific gravity has a direct and very close relation to density, and for any one body relatively to different conditions of itself, the one term is so entirely dependent upon the other that the terms, in such limited relationship, may be considered almost synonomous; in fact, in such a case, the specific gravity measures the density; and *vice versâ*; but if the comparison be made with another body, composed of a different kind or variety of matter, the same necessary inter-dependence no longer holds good; because the equal bulk of the second body may have (and the variety of matter being different, it *will* almost certainly *have*) a different atomic weight; and consequently although the first body may be in its most dense, and the second in its least dense condition, the specific gravity of the second may be nevertheless greater than that of the first. The intensity of the gravitating influence at the surface of the earth may be measured by the velocity acquired or the space passed through in a definite time by a falling body. The law which governs and regulates the motion and progress of a body so falling is the law of gravitation; and the conditions which accelerate or retard the descent of a falling body have been investigated with considerable attention and care. One of the facts conclusively ascertained is, as previously stated, that neither an increase in the density of a body through contraction, nor decrease

through expansion of the volume, nor yet an addition to the bulk (that is, an addition to the quantity of matter contained in the body) makes any difference as to rapidity in the descent of the falling body; the motion is neither accelerated nor retarded; the velocity is the same. But: what if there be a difference in the atomic weights between two bodies? will that make no difference in the relative velocity of their descent? Supposing there is a considerable difference between the atomic weights of two solid bodies, and that both of them are allowed to fall from the same definite height to the ground:—will they reach the ground in precisely the same time? Surely this is a question of great importance to which a conclusive answer will have been obtained, by very careful and reliable experiment. Let us see. An experiment which is reputed to have decided this question, was tried some considerable time since, and is supposed to have been frequently repeated. It is known as the 'Guinea and Feather experiment.'

Lardner's Natural Philosophy, page 108.

(236) "*Guinea and feather experiment.*—Let a glass tube AB, of five or six feet in length, be closed at one end B, and supplied with an air tight cap and stop cock at the other end A. The cap being unscrewed, let small pieces of metal, cork, paper and feathers be put into it, the cap screwed on, and the stop cock closed. Let the tube be rapidly inverted, so as to let the objects included fall from end to end of the tube. It will be found that the heavier objects, such as the metal, will fall with greater, and the lighter with less speed, as might be expected. But that this difference of velocity in falling is due, not to any difference in the operation of gravity, but to the resistance of the air, is proved in the following manner. Let the stopcock be screwed upon the plate of an air pump, the cock being open, and let the



tube be exhausted. Let the cock then be closed, and unscrewed from the plate. On rapidly inverting the tube, it will be found that the feathers will be precipitated from end to end as rapidly as the metal, and that in short, all the objects will fall together with a common velocity."

This is the experiment and such the result obtained, which is generalised, and applied as follows:—

Page 109. "*Weight of bodies proportional to their quantities of matter.*—Since the attraction of the earth acts equally on all the the component parts of bodies, and since the aggregate forces produced by such attraction constitute what is called the weight of the body, it is clear that the weights of bodies must be in the exact proportion of the number of particles composing them, or of their quantity of matter. Hence, in the common affairs of commerce, the quantities of bodies are estimated by their weights. It will appear, hereafter, that the weight of a body, or the force with which it is attracted to the surface, is slightly different in different places upon the earth; but this is a point which need not be insisted on at present. At the same place the weights are invariably and exactly proportional to the quanties of matter composing the bodies. If one body have double or triple the weight of another, it will have double or triple the quantity of matter in the other." We have herein a decided assumption that the elementary particles of matter are of equal weight; whatever other differences between various kinds of matter there may be, no distinction is made in respect to this important property; the elementary particles of matter, whatever descriptions or varieties of matter may be contained in the various substances known to us, are all of equal weight; and this is not merely an assumption taken as a basis for argument, nor is it a theory offered for consideration, but a positive statement as of a fact manifestly and conclusively established. "It is clear that the

weights of bodies must be in the exact proportion of the number of particles composing them or of the quantity of matter." "If one body have double or triple the weight of another, it will have double or triple the quantity of matter in the other."

Let us see how this statement harmonizes with the facts of chemistry.

Fowne's Manual of Chemistry, page 189.

(3) *Law of Equivalents.*—" It is highly important that the subject now to be discussed should be completely understood. Let a substance be chosen whose range of affinity and powers of combination are very great, and whose compounds are susceptible of rigid and exact analysis; such a body is found in oxygen, which is known to unite with all the elementary substances, with the single exception of fluorine. Now, let a series of exact experiments be made to determine the proportions in which the different elements combine with one and the same quantity of oxygen, which for reasons hereafter to be explained, may be assumed to be 8 parts by weight; and let these numbers be arranged in a column opposite the names of the substances. The result is a table or list like the following, but of course much more extensive when complete :

Oxygen	8
Hydrogen	1
Nitrogen	14
Carbon	6
Sulphur	16
Phosphorus	32
Chlorine	35.5
Iodine	127
Potassium	39
Iron	28
Copper	31.7
Lead	103.7
Silver	108
&c. &c	

Now the law in question is to this effect :—If such

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numbers represent the proportions in which the different elements combine with the arbitrarily fixed quantity of the starting substance, the oxygen; they also represent the *proportions in which they unite among themselves*, or at any rate bear some exceedingly simple ratio to these proportions."

(Page 193. *Combination by volume.*)

"The ultimate reason of the law in question (*combination by volume*) is to be found in the very remarkable relation established by the hand of Nature between the specific gravity of a body in the gaseous state and its chemical equivalent; a relation of such a kind that quantities by weight of the various gases expressed by their equivalents, or in other words, quantities by weight which combine, occupy under similar circumstances of pressure and temperature either equal volumes, or volumes bearing a simple proportion to each other." "If both the specific gravity and the chemical equivalent of a gas be known, its equivalent or combining volume can be easily determined, since it will be represented by the number of times the weight of an unit of volume (the specific gravity) is contained in the weight of one chemical equivalent of the substance. In other words, the equivalent volume is found by dividing the chemical equivalent by the specific gravity."

If we consider the elementary atoms of the chemist to be the elementary particles of matter, then, it is quite evident, that these results, of very numerous carefully conducted chemical experiments, entirely disagree with the deductions from the guinea and feather experiment previously detailed; because the information furnished us by these experiments is that the weight of a body consists in the atomic weight of its elementary particles multiplied into the number of those particles; or in other words, the atomic weight of that particular description of matter of which the body consists multiplied into the quantity thereof.

Now we have seen that the physicist as represented by Dr. Lardner, has unlimited confidence in these generalizations and conclusions which based upon the guinea and feather experiment are considered as the exposition of an established law. Does the chemist show equal confidence in the atomic theory, and in those experiments upon the results of which its title and claim to confidence are founded.

Fowne's Manual of Chemistry. Page 200. "The theory in question (the atomic theory) has rendered great service to chemical science, &c., &c." "At the same time, it is indispensable to draw the broadest possible line of distinction between this, which is at the best but a graceful, ingenious, and in its place, useful hypothesis, and those great general laws of chemical action which are the pure and unmixed result of inductive research."

"Note.—The expression atomic weight is very often substituted for that of equivalent weight, and is, in fact, in almost every case to be understood as such; it is perhaps better avoided."

So that the atomic theory is not only considered inconclusive, but it is thought proper to caution the student to look upon it with a sort of distrust, as being at best, only a graceful and ingenious hypothesis. To show how far we dissent from this teaching on the subject, we will express our belief that the atomic theory is, and has been for some time past, *virtually a demonstrated theorem*; and, as such, shown to be a *compound fact,—the* great fundamental *fact* upon which the structure of chemical science rests. It is true, it has not been, as yet, formally demonstrated; but that is, apparently, because no one has taken into consideration the possible consequences direct and indirect of leaving a science such as chemistry without any demonstrated and acknowledged basis. One of the consequences is the caution given to the student, as above. There might be, however, an objection to admitting the atomic theory (and the law of combining equivalents) as a demonstrated

theorem, side by side with that (so called) law of natural philosophy (previously stated) based on the guinea and feather experiment; because if understood in the usual sense, one of these laws evidently to some extent contradicts the other. It therefore seems desirable to give a little more particular attention to the g. and f. experiment as recorded by Dr. Lardner. We will first consider for a moment the circumnstances under which the experiment has probably been tried. What were the expectations and what the express object of the experimenter? The experimenter has been previously informed, perhaps, as to what the result will be, and is consequently prejudiced or predisposed to expect and to accept such result. The express object of the experiment in the first instance was evidently to ascertain the effect of removing the resistance of the air to the descent of a falling body; not to compare the relative velocities of descent in substances differing in kind. Taking the length of the glass tube at five feet and one third, and assuming that there may be some difference in the velocities of descent, what might be expected to take place? The space fallen through by a weight in a second has been established as $16\frac{1}{12}$ feet; lead or iron being the material generally used for the weight. The pieces of metal, therefore, might be expected to fall from end to end of the tube in about one third of a second; now supposing a very considerable difference, and that the pieces of feather or cork were to take more than half a second (or say even two-thirds of a second) it might be possible, but it would be by no means very easy, to observe such a difference; and this is supposing that all the objects in a glass tube of only 4 inches bore, which has to be inverted, start fairly together and do not come into contact with the side of the tube. The experiment, as to its affording any precise and reliable information about the velocities of bodies falling in vacuo, seems to us scarcely worthy of consider-

ation. If those who object to the atomic theory and the teaching of chemistry have nothing more reliable to oppose to it than the result of such an experiment, the chemist may fairly claim demonstration, on his side, and to have the general result of his numerous experiments admitted to the place of an established fact. But can we not get at what would necessarily be the result of the G. and F. experiment, if it were to be properly tried, by deducing the result from the results of reliable experiments which have already been tried and recorded? The specific gravities of the various metals have been carefully determined. A cubic inch of (cast) iron weighs 4.17 oz. A cubic inch of (cast) lead weighs 6.37 ozs. Now if we take a cubic inch of each of these metals, and connecting the two weights by a fine line, suspend them in an Attwood's machine by passing the connecting line over the grooved wheel,—we can say with certainty what will happen; viz., the lead weight which is more than 2 ozs. heavier than the other will descend with a considerable and a continually accelerated velocity; and, in doing so will raise the iron weight with an equal velocity. If the two weights are now detached and allowed to fall from the same height to the ground--will they descend with an equal velocity? No doubt they will; because in doing so the equally rapid descent of the heavier weight will represent a larger quantity of effect exactly proportionate to the preponderance of weight. Does this decide the question? Dr. Lardner's deduction is that the quantity of matter or number of particles contained in the lead is greater than in the iron, and *therefore* both of them descend with the same velocity; but, taking the atomic theory, are we thereby taught that an atom of lead is of precisely the same weight as an atom of iron, or of gold, or of potassium? Does the cubic inch of lead contain a greater number of elementary particles of lead than the cubic inch of iron contains of the elementary particles of iron? It is evident that the deduction of Lardner becomes

or includes a primary definition of matter; in other words it involves the positive statement (corollary), that if (a) and (b) represent two distinct varieties of matter, and the combining equivalent (or atomic weight) of (a) is twice that of (b), the elementary atom of (a) contains twice the quantity of primary matter (*i.e.* of matter in a more simple and elementary form or condition) contained by the elementary atom of (b). If the elementary atoms are of the same size, then that of (a) must have twice the density compared with that of (b). The important distinction herein defined is that gravity is not a property of which one variety of matter possesses more or less than others; but belongs to a primary form or condition of matter, and that a fundamental difference between all those varieties of matter known to us, is that the elementary atom of any one variety is compounded of a greater or of a lesser quantity of primary matter, than the elementary atom of any one of the other varieties. We do not say that the conclusion thus arrived at is unsound; on the contrary, we are strongly of opinion that such conclusion may be demonstrated and established, and with such interpretation and definition the law stated by Lardner and the atomic theory harmonize perfectly; but it does not follow that the admission of a hasty generalization based on a rough and inconclusive experiment to stand as a part of the *national science* is justified because it may eventually appear that the generalization was not *in fact* false. We are under the impression that Dr. Lardner himself would have hesitated to accept, and might very possibly have rejected the *corollary* to the proposition so positively stated.

It has been said—by a writer whose reasoning powers cannot be lightly esteemed, and whose opinions and statements, considering the general state of knowledge and other circumstances at the time in which he wrote, are certainly entitled to respect—that 'a little knowledge is a dangerous thing'; the saying may be amplified, and it may be said, with no less truth, that a good deal of knowledge is a dangerous thing,—if that knowledge is of an uncertain, unsound and disorderly nature, containing sound and unsound knowledge,—truth and untruth—good and evil—mingled indiscriminately together; and it may be also said—that men possessed of a little knowledge, or of unsound and uncertain knowledge, are dangerous; dangerous to themselves and to each other; unless they be controlled by that superior knowledge which is sound and certain.

Many persons think that human knowledge is now far in advance of what it has been at any former time. This is an opinion which may have, and very probably has been frequently, entertained at earlier periods of the world's history. Many educated persons suppose that civilization and science have progressed so much and are now so firmly established, that no general catastrophe or even serious reverse is any longer to be feared; a nation here and there may fall behind, and mistakes may sometimes be made; but, in a general or universal sense, civilization is safe, and must continue to progress with accelerated rapidity. The grounds upon which such reliance is based appear to be somewhat indefinite. Some put their trust in the extended area of civilization and the general diffusion of education; some have faith in the multitude of books; and others feel a happy assurance in the power of railways, steamboats and telegraphs, to ward off and avert any dangers that civilization may be exposed to.

We do not wish to exaggerate the signs which appear to us to indicate particular danger, and we certainly do

not wish to create any unnecessary alarm; but we will conclude with one word of caution, and will say to those who think, because——we have lived for long in a time of calm, because——the mutterings of danger occasionally heard have ceased, and the signs of storm, which from time to time have shown themselves, have again passed away and the heavens are still serene, that therefore——we may live on in careless security, and that prudence and precaution are needless; to such persons we say....take care, it is unsafe.